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TIMO HASSINEN

Studies on Coherence and Purity of Electro-magnetic Fields

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ABSTRACT

This thesis contains theoretical studies on coherence and purity of electromagnetic fields. The concept of purity concerns the invariance of certain physical field properties under superposition. The cross-spectral purities of vector fields and the Stokes parameters are discussed. These concepts are equivalent to the spectral invariance of one Stokes parameter in Young's interference experiment and lead to the factorization of the corresponding two-point Stokes parameter. Additionally, the polarization purity, or invariance, in Young's experiment is studied in the space–time and space–frequency domains. It is shown to result in the separation of polarization and spatial modulation in the cross-spectral density matrix. Moreover, the differences of polarization purity between the two domains are discussed.

The spectral invariance is investigated also on propagation into the far zone in the case of vector fields emitted by a certain class of sources. For such sources, a so-called scaling law is derived. This law leads to the normalized far-field spectrum being the same in every direction and equal to the normalized source spectrum. Furthermore, the Hanbury Brown–Twiss experiment is analyzed in the case of electromagnetic waves. It is shown that for vector fields that obey Gaussian statistics the degree of coherence for electromagnetic fields fully describes the correlations of intensity fluctuations.

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Preface

First of all, I am most indebted to my supervisors Jani Tervo and Ari T. Friberg for guidance and providing research topics during all these years. Jani introduced me to the world of coherence optics and gave me a possibility to work in the wave-optics group even before my PhD studies, and Ari has generously shared the wisdom he has acquired from all over the world.

I am grateful to the former and present heads of the Department of Physics and Mathematics, Timo Jääskeläinen, Pasi Vahimaa, and Markku Kuittinen, for the opportunity to work in the university. I also want to thank my co-authors Tero Setälä and Jari Turunen for their contribution to the articles that made this thesis possible, and the Emil Aaltonen Foundation for financial support. The valuable comments from the reviewers Gemma Piquero Sanz and Sergei Popov are highly appreciated.

During my PhD studies I was privileged to work half a year in Rome. I express my sincere gratitude to Massimo Santarsiero who made this possible and gave access to his lab in Roma Tre University. I also want to thank the Finnish Cultural Foundation and the Magnus Ehrnrooth Foundation for the personal grants they provided for the research visit.

I am much obliged to the whole staff of the department for all the help and assistance. Special thanks go to Kimmo for all the discussions on and off the subject (mostly off), and to the Ultimate squad for making me run approximately once a week.

Finally, I want to thank my parents Anja and Veli for all the love and support, and my siblings Jukka and Kati just for being there. Last but not least, I want to express my warmest gratitude to Martta for all the encouragement and sharing her life with me.

Joensuu, November 7, 2013 *Timo Hassinen*

LIST OF PUBLICATIONS

This thesis consists of the present review of the author's work in the field of electromagnetic coherence theory and the following selection of the author's publications:

I T. Hassinen, J. Tervo, and A. T. Friberg, "Cross-spectral purity of electromagnetic fields," *Opt. Lett.* **34,** 3866–3868 (2009).

II T. Hassinen, J. Tervo, and A. T. Friberg, "Cross-spectral purity of the Stokes parameters," *Appl. Phys. B* **105,** 305–308 (2011).

III T. Hassinen, J. Tervo, and A. T. Friberg, "Purity of partial polarization in the frequency and time domains," *Opt. Lett.* **38,** 1221–1223 (2013).

IV T. Hassinen, J. Tervo, T. Setälä, J. Turunen, and A. T. Friberg, "Spectral invariance and the scaling law with random electromagnetic fields," *Phys. Rev. A* **88,** 043804 (2013).

V T. Hassinen, J. Tervo, T. Setälä, and A. T. Friberg, "Hanbury Brown–Twiss effect with electromagnetic waves," *Opt. Express* **19,** 15188–15195 (2011).

Throughout the overview, these papers will be referred to by bold Roman numerals.

AUTHOR'S CONTRIBUTION

The publications selected for this dissertation are original research articles on the purity and coherence of electromagnetic fields.

The author has performed the theoretical calculations and written the manuscripts in papers **I–III**. In paper **IV** the author has made the theoretical calculations for Sections III and IV, and partly written the manuscript. The author has participated in the theoretical calculations and the writing of the manuscript in paper **V**.

All papers have been finalized in significant cooperation with the co-authors.

Contents

1 Introduction

Sunlight makes the life on Earth possible as we know it. It brings warmth from millions of kilometers away and gives energy to plants and trees, providing food for herbivores and whole food chains. The Sun has also fascinated and inspired mankind through history. In many cultures and religions it has been seen as a divine force or other mystical power but later science has provided us with better understanding of the true nature of sunlight. Nowadays we see light as electromagnetic radiation that is visible to the human eye and has properties of waves and particles alike.

However, what is not visible to the naked eye is that sunlight, as well as other natural or artificial light, contains more or less random fluctuations. This randomness originates from the creation process of radiation and from the random fluctuations of the transmitting media. In the generation of light an excited atom randomly emits a ″light particle″ called photon. In a realistic physical source a multitude of adjacent atoms of this kind emit photons creating a stream of randomly fluctuating radiation. It is possible, however, that the atoms in the source are somehow correlated, i.e., dependent on each other, which also leads to correlation between the emitted photons. Thus, the electromagnetic fields at different positions or time instants within the photon stream may be correlated, allowing them to interfere with each other. This means that fields can superimpose constructively or destructively and form a fringe pattern when merged in an interference experiment. The section of optics research that investigates these field correlations and different phenomena arising from them is called the optical coherence theory.

In this chapter the historical development of the classical coherence theory is discussed from the perspective of both the scalar and the electromagnetic fields. The motivation for the research and the research goals are addressed, and the contents of each chapter are outlined.

1.1 HISTORICAL BACKGROUND

Prior to the general acceptance of the dualistic nature of light both the particle and the wave theories had their supporters. Before the 19th century the majority of scientists, including Newton, hypothesized that light consists of particles, mainly due to its rectilinear propagation [1]. In the beginning of the 19th century, however, the well-known interference experiment performed by Young [2] proved the wave nature of light as the particle theory could not explain diffraction or interference. Young's experiment together with the research done by Hooke, Huygens, and Fresnel formed a strong basis for the reign of the wave theory of light that eventually replaced the particle theory [3].

Later in the 19th century optical wave theory was combined with the theory of electromagnetism by Maxwell [4] who put together a set of partial differential equations describing the behavior of electric and magnetic fields. Based on these Maxwell equations the speed of an electromagnetic wave was calculated to be equal to the speed of light, a fact that together with an experimental proof by Hertz showed light to be electromagnetic radiation [1]. Additionally, in 1887 Hertz discovered the photoelectric effect, a phenomenon in which incoming light causes material to emit electrons. Such an event was not explainable by the wave theory, so it took almost two decades, the comeback of the particle theory, and one of the greatest scientists of all time to explain the phenomenon. The photoelectric effect was described by means of light particles, later called photons, in 1905 by Einstein [5] whose work was based on the earlier studies of Planck. Both scientists later received Nobel prizes for their efforts. Since there exist phenomena that are not explainable in terms of the wave or particle theory alone, light today is generally considered to be dualistic.

The first studies on optical coherence were performed in the late 19th century by Verdet [6] and Michelson [7–9], another Nobel laureate. Verdet carried out coherence measurements on sunlight, whereas Michelson introduced the concept of visibility to characterize an interference fringe pattern and used interferometric means to calculate the sizes of different astronomical objects. The next remarkable advances of coherence theory came in the 1930s. Wiener [10] introduced the correlation functions as mathematical tools to analyze coherence at two space–time points and noticed that there is a Fourier-transform relation between the autocorrelation function and the spectral density. A similar conclusion, today known as the Wiener–Khintchine theorem, was achieved independently a few years later by Khintchine [11]. In the same decade Zernike [12] presented the first satisfactory measure of correlation as he introduced the degree of coherence that was related to the visibility of interference fringes. He also simplified the interesting result which van Cittert [13] had derived earlier. This result, nowadays known as the van Cittert–Zernike theorem, leads to a conclusion that the far-field of a distant incoherent source is mainly coherent under certain conditions.

In the 1950s Wolf [14, 15] generalized the correlation functions to handle both the temporal and spatial separation between the investigated fields, and Hanbury Brown and Twiss [16,17] performed their famous experiment. The experiment dealt with the intensity correlations between two beams of light and it was shown that for thermal sources such correlations are related to the degree of coherence. In 1961 Mandel [18] introduced the concept of cross-spectral purity that relates the spectral invariance of fields on superposition to the factorization of the complex degree of coherence. Mandel and Wolf [19] analyzed the concept further in 1976, together with a proper introduction of the spectral degree of coherence that was already employed informally in some earlier publications [20–22]. Until then the coherence properties of light had been studied almost exclusively in the space–time domain but, due to its physical convenience, e.g., in scattering problems, the space–frequency approach was gaining ground. This led to the formulation of spectral coherence theory by Wolf [23–26] in the 1980s. Wolf [27] investigated also the spectral invariance of light on propagation and derived a scaling law ensuring that the normalized spectrum of light from a quasi-homogeneous source is directionally invariant in the far zone.

In all of the above-mentioned works the analysis was restricted to scalar fields, neglecting the polarization which is one of the fundamental properties of light. To take into account the polarization phenomena that are connected to the correlations between the orthogonal field components in a single space–time point, light has to be modeled as a vector-valued fluctuating field. Some of the early studies on polarization were performed in the middle of the 19th century by Stokes [28] who described the polarization state of light through four parameters. Seven decades later Wiener [10] presented an alternative to these Stokes parameters by describing the polarization matrix. The concept was more thoroughly examined in 1959 by Wolf [29] who also introduced the degree of polarization to distinguish partially polarized fields from unpolarized or fully polarized ones. In 2002 the degree of polarization was generalized from two- to three-dimensional situation by Setälä *et al.* [30, 31]. Later, also other measures of the polarization of three-component fields have been proposed [32].

The correlations of vector field at two space–time points were first studied by Wolf [33] in 1954 using the matrix formalism, and the first measure for electromagnetic coherence was suggested by Karczewski [34, 35] in 1963. Four decades later Karczewski's direct extension of the scalar degree of coherence was reintroduced by Wolf [36], this time in the frequency domain. However, fueled by the inability of Karczewski's degree of coherence to fully describe the coherence properties of vector fields, another electromagnetic degree of coherence was proposed by Tervo *et al.* [37] in 2003. Since then, due to the increased interest towards the electromagnetic correlation phenomena, also other measures of coherence have been suggested [38–40]. Moreover, the Stokes parameters have been generalized to describe the correlations at two positions in space [41, 42].

Additionally, the quantum theory of optical coherence was pioneered in 1963 by Glauber [43], who was later awarded a Nobel prize for his efforts. Since then, this theory has become an important field of study in modern optics [44–47]. However, in this thesis we discuss only classical coherence phenomena that do not require quantum treatment.

1.2 MOTIVATION AND GOALS

With the development of nanophotonics the need to analyze the behavior of light in matter containing subwavelength-scale features has grown. For such an analysis, as well as for examining optical near fields, the scalar theory is generally insufficient and a rigorous electromagnetic approach has to be employed. As the correlation phenomena related to the vectorial nature of light began to attract more attention due to the simultaneous introduction of two different coherence measures only a decade ago, there still is a lot to learn about optical electromagnetic fields. The main goal of this thesis is to gain more insight into the vectorial nature of light by examining various coherence phenomena related to partially coherent and partially polarized fields.

The research performed in this thesis can be divided into three parts. The first part contains studies related to the purity of electromagnetic fields, inspired by Mandel's concept of cross-spectral purity [18] and the research of Gori *et al.* [48] on different classes of unpolarized beams. The purpose of these studies, published in papers **I–III**, is to derive conditions that result in the invariance of the spectrum or the polarization state in Young's interference experiment for vector fields. In the second part the prerequisites for the spectral invariance on propagation in free space are considered for electromagnetic fields. This research, performed in the spirit of Wolf's scalar studies [27], was published in paper **IV**. The third topic of the thesis is the Hanbury Brown–Twiss (HBT) effect that connects the intensity fluctuations of thermal light to its degree of coherence in the scalar case. The HBT experiment was analyzed with vector fields already some years ago [49–51] but, due to the employment of Wolf's electromagnetic degree of coherence [36] in them, there is a need to examine the situation also from another point of view. Hence, the aim of paper **V** is to investigate the HBT effect with vector fields by utilizing the degree of coherence for electromagnetic fields by Tervo *et al.* [37] and to discuss possible differences to earlier results.

1.3 OUTLINE

The rest of the thesis is organized as follows. The fundamentals of the scalar coherence theory are presented in Chapter 2 and extended to the case of vector fields in Chapter 3, together with a brief overview of polarization. In Chapter 4 the concept of cross-spectral purity is explained and the related results in papers **I–III** are discussed. Chapter 5 deals with the spectral invariance on propagation by first examining scalar fields and then making an extension to the electromagnetic theory based on paper **IV**. The HBT experiment is analyzed within scalar and vector theories, together with a discussion according to paper **V**, in Chapter 6. Finally, in Chapter 7 the thesis is concluded with the summary of main results and an outlook for future work.

2 Fundamentals of scalar coherence theory

The coherence properties of light are usually studied within the framework of the classical second-order coherence theory [52]. It provides adequate means to understand phenomena arising from the field correlations at two space–time points and is the backbone of optical coherence theory in general. In order to avoid misunderstandings, it should be noted, however, that in the context of quantum optics the analogous correlations are said to be of first-order instead [43, 52].

Throughout this chapter light is represented by scalar fields or scalar functions. Even though such an approach is often valid, it neglects some important phenomena characteristic of vector fields such as partial polarization (see Section 3.1). Most of the subjects discussed here in the context of scalar optics are extended for vector fields in Chapter 3.

In the following sections the elementary tools needed to analyze optical fields mathematically are introduced. The scalar coherence theory in the space–time domain is formulated and a method, namely Young's interference experiment [2], is presented to measure coherence properties. Further, the famous van Cittert–Zernike theorem [12, 13] is recalled and the scalar coherence theory is extended to the frequency domain using the Wiener–Khintchine theorem [10, 11]. Finally, the chapter is closed with a brief look into the propagation of scalar partially coherent fields in free space.

2.1 BASIC CONCEPTS

In the scalar coherence theory the light at a position $\mathbf{r}$ at time t is represented by a scalar field $E_{\text{re}}(\mathbf{r}, t)$, that satisfies the wave equation in vacuum. Here the subscript "re" is used to emphasize that the field is real-valued. Even though E_{re} is real, in classical coherence theory it is practical to express it as a complex random function $E(\mathbf{r}, t)$ using the complex analytic signal representation by Gabor [53]. This is mainly done to simplify mathematical analysis, but the complex analytic signal also provides a connection between the classical and quantum theories of optical coherence [52, 54]. In essence, the complex analytic signal is an unambiguous complex function that has the same normalized Fourier spectrum as the original field at the positive frequencies. Throughout this thesis we use the complex analytic signal representation for the fields.

As optical fields vibrate rapidly it is not possible to measure their absolute phases accurately. Moreover, the photon emissions from light sources and the fluctuations of the transmitting media are generally random which results in randomly fluctuating electromagnetic fields. These physical facts lead to the modeling of light by means of statistical optics where field quantities are considered as random functions or processes and not as deterministic ones. It is nearly impossible to tell exact information about fields so statistical concepts, such as expectation values and probability densities, have to be used. The probability density of $E(\mathbf{r}, t)$, denoted by $p(E, \mathbf{r}, t)$, gives information about the possible values of E and their probabilities. The expectation value of E, on the other hand, is defined statistically as the weighted average

$$\langle E(\mathbf{r}, t)\rangle = \int_{\mathbb{C}} p(E, \mathbf{r}, t) E \, \mathrm{d}E, \tag{2.1}$$

where integration is performed over the complex plane $\mathbb{C}$. An alternative approach is to think that $E(\mathbf{r}, t)$ has a discrete set or an ensemble of possible values, i.e., realizations $E_m(\mathbf{r}, t)$, at a time instant t. In that case the expectation value is written as

$$\langle E(\mathbf{r}, t)\rangle = \lim_{M\to\infty} \frac{1}{M} \sum_{m=1}^{M} E_m(\mathbf{r}, t). \tag{2.2}$$

The probability density functions of different orders create a hierarchy where each function of higher order contains all the information of the lower-order functions. This leads to a hierarchy of correlation functions that are used in the coherence theory.

Many light sources operate continuously and emit light that has the same statistical parameters at all reasonable time instants. Such light is called statistically stationary and includes, e.g., sunlight and continuous laser beams. Laser pulses, on the other hand, are an explicit example of non-stationary light. Moreover, if all realizations of the random function E have the same statistical parameters, the function is called ergodic. For such functions the ensemble average and time average coincide, and it is possible to use either one in the analysis. In this thesis we consider the investigated fields to be both stationary and ergodic.

2.2 COHERENCE IN SPACE–TIME DOMAIN

Let us now consider second-order correlations of a stationary and ergodic field between two space–time points. For that the mutual coherence function [15]

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) = \langle E^*(\mathbf{r}_1, t) E(\mathbf{r}_2, t+\tau) \rangle, \tag{2.3}$$

where the asterisk denotes the complex conjugate, is introduced. The time difference $\tau = t_2 - t_1$ is used here due to the stationarity as only the difference between the absolute observation times matters. To turn Γ into a more quantitative physical measure, it is normalized as

$$\gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) = \frac{\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau)}{[I(\mathbf{r}_1) I(\mathbf{r}_2)]^{1/2}}, \tag{2.4}$$

where $I(\mathbf{r}_j) = \Gamma(\mathbf{r}_j, \mathbf{r}_j, 0)$, with $j \in (1, 2)$, are the time-averaged intensities at points $\mathbf{r}_j$. The normalized function $\gamma(\mathbf{r}_1, \mathbf{r}_2, \tau)$ is called the complex degree of coherence for reasons explained in the next section.

2.3 YOUNG'S INTERFERENCE EXPERIMENT

Young's interference experiment [2], named after English scientist Thomas Young, is perhaps the single measurement setup that has had the biggest impact on the development of coherence theory [3]. The setup, illustrated in Fig. 2.1, consists of a non-transparent screen $\mathcal{A}$ with two pinholes at positions $\mathbf{Q}_1$ and $\mathbf{Q}_2$ in it, and an observation screen $\mathcal{B}$ placed after it. The distance between these screens is large compared to wavelength of light. The screen $\mathcal{A}$ is illuminated with quasi-monochromatic light which means that the effective bandwidth $\Delta\omega$ of light is much smaller than its mean frequency $\bar{\omega}$, i.e.,

$$\frac{\Delta\omega}{\bar{\omega}} \ll 1. \tag{2.5}$$

Here $\omega = 2\pi c_0/\lambda$, where c_0 is the speed of light in vacuum and λ is the wavelength, is the angular frequency. To get the desired results, the investigated time differences between optical paths have to be small compared to the coherence time. Moreover, the pinholes have to be small enough that the field within them is constant but large enough that the diffraction from the edges of pinholes may be neglected.

With the above characteristics and the fields in the pinholes acting as secondary sources the field at the screen $\mathcal{B}$ in the scalar case is written as [52]

$$E(\mathbf{r}, t) = K_1 E(\mathbf{Q}_1, t - t_1) + K_2 E(\mathbf{Q}_2, t - t_2), \tag{2.6}$$

where $\mathbf{r}$ is a position at the observation screen and $t_j = R_j/c_0$, with $j \in (1, 2)$, are the times in which light travels from the pinholes to the observation point. The distances between the pinholes and $\mathbf{r}$ are denoted by R_j, and $K_j \approx -\mathrm{i}A/(\bar{\lambda}R_j)$, where $\bar{\lambda}$ is the mean wavelength, are purely imaginary constants [1, 52, 54] that depend on the size of the pinholes A and on the geometry. The averaged intensity at $\mathcal{B}$ is now of the form [52, 54]

$$\begin{aligned} I(\mathbf{r}) = |K_1|^2 I(\mathbf{Q}_1) + |K_2|^2 I(\mathbf{Q}_2) + 2|K_1||K_2|\sqrt{I(\mathbf{Q}_1)I(\mathbf{Q}_2)} \\ \times |\gamma(\mathbf{Q}_1, \mathbf{Q}_2, \tau)| \cos[\alpha(\mathbf{Q}_1, \mathbf{Q}_2, \tau) - \bar{\omega}\tau], \end{aligned} \tag{2.7}$$

where $\alpha(\mathbf{Q}_1, \mathbf{Q}_2, \tau) = \arg\{\gamma(\mathbf{Q}_1, \mathbf{Q}_2, \tau)\} + \bar{\omega}\tau$ is a slowly varying phase term, and the terms $|K_j|^2 I(\mathbf{Q}_j)$, with $j \in (1, 2)$, correspond to the intensities on the observation screen when only pinhole $\mathbf{Q}_j$ is open. It should be noted that here the time difference τ is a function of the position $\mathbf{r}$ at the observation screen. This equation is called the general interference law for stationary quasi-monochromatic fields [54]. The changes in the intensity are governed by the term $\bar{\omega}\tau$ when $\mathbf{r}$ is slightly altered, leading to almost sinusoidal interference pattern in the vicinity of the observation point.

The standard method to characterize interference patterns is to employ the visibility of interference fringes, a measure introduced by Michelson in 1890 [7]. Visibility is defined by means of the maximum and minimum intensities, I_{max} and I_{min}, at the neighborhood of $\mathbf{r}$ as

$$V_{\text{I}}(\mathbf{r}) = \frac{I_{\text{max}}(\mathbf{r}) - I_{\text{min}}(\mathbf{r})}{I_{\text{max}}(\mathbf{r}) + I_{\text{min}}(\mathbf{r})}. \tag{2.8}$$

In Young's interference experiment the visibility can thus be written

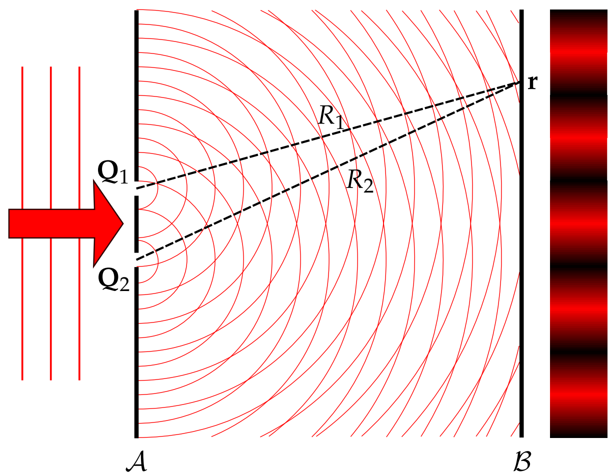

Figure 2.1: Notations for Young's interference experiment. Light illuminates the screen $\mathcal{A}$ with pinholes at positions $\mathbf{Q}_1$ and $\mathbf{Q}_2$, and interference fringes are formed on the observation screen $\mathcal{B}$. The distances from the pinholes to the observation point $\mathbf{r}$ are denoted by R_1 and R_2.

as

$$V_{\mathrm{I}}(\mathbf{r}) = \frac{2\sqrt{I(\mathbf{Q}_1)I(\mathbf{Q}_2)}}{I(\mathbf{Q}_1) + I(\mathbf{Q}_2)} |\gamma(\mathbf{Q}_1, \mathbf{Q}_2, \tau)|, \tag{2.9}$$

where the first term equals unity when the light intensities in the pinholes are equal. It is now clear that the modulus of γ is related to the ability of light to form interference fringes. Hence, based on the definition of coherence by Zernike [12], $|\gamma(\mathbf{Q}_1, \mathbf{Q}_2, \tau)|$ is called the degree of coherence. It is bounded by zero and unity, where $|\gamma| = 0$ corresponds to completely incoherent and $|\gamma| = 1$ completely coherent field. Otherwise the field is called partially coherent. For incoherent fields, no interference fringes are formed as there is no correlation between the fields in the pinholes. The argument of γ, on the other hand, is associated with the locations of the fringes [52].

Coherence can be divided to spatial and temporal coherence. Within spatial coherence the field correlations at two points in space at one time instant are examined. Spatial coherence is characterized by the spatial degree of coherence $\gamma(\mathbf{r}_1, \mathbf{r}_2, 0)$. As for temporal coherence, the normalized autocorrelation function $\gamma(\mathbf{r}, \mathbf{r}, \tau)$ is used and field correlations between two time instants at a single position are investigated. Also, if the space–time points $(\mathbf{r}_1, t_1)$ and $(\mathbf{r}_2, t_2)$ coincide, $\gamma(\mathbf{r}, \mathbf{r}, 0) = 1$ meaning that the field is self-coherent, i.e., completely correlated with itself. Spatial and temporal coherence are generally intertwined but there exists a class of fields for which the coherence properties separate to spatial and temporal parts. Fields of this kind are called cross-spectrally pure [18] and we take a closer look at them and their properties in Chapter 4.

2.4 VAN CITTERT–ZERNIKE THEOREM

In this section we recall the well-known van Cittert–Zernike theorem formulated by van Cittert [13] and Zernike [12] in the 1930s. The theorem deals with the spatial coherence of a field emitted by a planar source that is spatially incoherent and quasi-monochromatic. According to it, the spatial complex degree of coherence for such a field can be expressed in the form [52]

$$\gamma(\mathbf{r}_1, \mathbf{r}_2, 0) = [I(\mathbf{r}_1)I(\mathbf{r}_2)]^{-1/2} \left(\frac{\bar{k}}{2\pi}\right)^2 \int_D I(\boldsymbol{\rho}') \frac{e^{\mathrm{i}\bar{k}(R_2-R_1)}}{R_1 R_2} \, \mathrm{d}^2\rho', \quad (2.10)$$

where $\bar{k} = \bar{\omega}/c_0$ is the mean wave number, R_1 and R_2 are the distances from source point $\boldsymbol{\rho}'$ to $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively, and integration is performed over the source area D.

According to Eq. (2.10) the spatial coherence properties of the field radiating from a planar, quasi-monochromatic and spatially incoherent source are governed by the intensity distribution of the source. Thus, a field emitted by an incoherent source is in general spatially partially coherent when it has traveled large distances. This has applications especially in astronomy as most astronomical sources are spatially incoherent. Besides earlier electromagnetic extensions [55–57], the van Cittert–Zernike theorem has also been recently reassessed [58] with the help of the Stokes parameters and the generalized Stokes parameters, quantities that will be introduced in Chapter 3.

2.5 COHERENCE IN SPACE–FREQUENCY DOMAIN

In Young's interference experiment quasi-monochromatic illumination has to be used to retain the connection between the field correlations in the pinholes and the visibility of interference fringes. The time-domain coherence theory is a practical tool in the analysis of such fields but to examine broadband light the spectral coherence theory is often more convenient.

In the space–frequency domain the cross-spectral density (CSD) function $W(\mathbf{r}_1, \mathbf{r}_2, \omega)$ is considered. It is defined as the Fourier transform of the mutual coherence function, i.e.,

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) e^{\mathrm{i}\omega\tau} \, \mathrm{d}\tau, \quad (2.11)$$

so it forms a Fourier-transform pair together with its inverse

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) = \int_0^{\infty} W(\mathbf{r}_1, \mathbf{r}_2, \omega) e^{-\mathrm{i}\omega\tau} \, \mathrm{d}\omega. \quad (2.12)$$

This pair can be seen as a generalization of the Wiener–Khintchine theorem. As for the original Wiener–Khintchine theorem, it states that the autocorrelation function $\Gamma(\mathbf{r},\mathbf{r},\tau)$ of a stationary random function forms a Fourier-transform pair with the spectral density $S(\mathbf{r},\omega) = W(\mathbf{r},\mathbf{r},\omega)$ [10,11].

The CSD function can also be expressed as a correlation function in the form $W(\mathbf{r}_1,\mathbf{r}_2,\omega) = \langle \mathcal{E}^*(\mathbf{r}_1,\omega)\mathcal{E}(\mathbf{r}_2,\omega)\rangle$, where $\mathcal{E}(\mathbf{r},\omega)$ are random complex amplitudes of monochromatic wave functions $\mathcal{E}(\mathbf{r},\omega)e^{\mathrm{i}\omega t}$ [23]. However, it should be pointed out that the functions $\mathcal{E}(\mathbf{r},\omega)$ are not Fourier transforms of the time-domain realizations $E(\mathbf{r},t)$, despite the Fourier-transform relation between $W(\mathbf{r}_1,\mathbf{r}_2,\omega)$ and $\Gamma(\mathbf{r}_1,\mathbf{r}_2,\tau)$ [24].

Similarly to the complex degree of coherence in Section 2.2, a measure for coherence in the space–frequency domain is defined as [19–22]

$$\mu(\mathbf{r}_1,\mathbf{r}_2,\omega) = \frac{W(\mathbf{r}_1,\mathbf{r}_2,\omega)}{[W(\mathbf{r}_1,\mathbf{r}_1,\omega)W(\mathbf{r}_2,\mathbf{r}_2,\omega)]^{1/2}}. \tag{2.13}$$

The modulus of this spectral degree of coherence, i.e., $|\mu|$, takes on values between zero and unity, where $|\mu| = 0$ and $|\mu| = 1$ correspond to complete incoherence and complete coherence, respectively. Even though there is a Fourier-transform connection given in Eqs. (2.11) and (2.12) between Γ and W, the complex degree of coherence and the spectral degree of coherence do not form a Fourier-transform pair but their relationship is more involved [59,60].

Complete coherence is also characterized by the spatial factorization of the correlation function [43,52]. In the space–time domain this means that if $\gamma(\mathbf{r}_1,\mathbf{r}_2,\tau) = 1$ for all τ and for all pairs of points $\mathbf{r}_1$ and $\mathbf{r}_2$ in some volume, the mutual coherence function factors as [52]

$$\Gamma(\mathbf{r}_1,\mathbf{r}_2,\tau) = V^*(\mathbf{r}_1)V(\mathbf{r}_2)e^{-\mathrm{i}\omega_0\tau}. \tag{2.14}$$

Here $V(\mathbf{r})$ is a function that depends only on position, and ω_0 is a constant. Respectively, complete coherence in some volume at frequency ω leads to the CSD function being of the form [52]

$$W(\mathbf{r}_1,\mathbf{r}_2,\omega) = \mathcal{V}^*(\mathbf{r}_1,\omega)\mathcal{V}(\mathbf{r}_2,\omega), \tag{2.15}$$

where $\mathcal{V}(\mathbf{r}, \omega)$ depends on the spectral density at $\mathbf{r}$. The inverse is also true, so if the spatial dependence of the correlation function separates, the field is completely coherent. The coherent fields of this kind are the basis of the coherent-mode representation that allows the expression of the CSD function as an incoherent superposition of coherent modes [52].

2.6 PROPAGATION OF PARTIALLY COHERENT FIELDS IN FREE SPACE

Optical measurements are often made long distances away from the source, at least compared to the wavelength of light, so it is valuable to know how the measured fields or other quantities of interest relate to those at the source. In this section the propagation of a scalar field from a planar secondary source to the far zone is examined in the spectral domain. The situation and used notations are illustrated in Fig. 2.2.

As $E(\mathbf{r}, t)$ satisfies the wave equation, the mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau)$ propagating through free space obeys the wave equations [15]

$$\nabla_j^2 \Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) = \frac{1}{c_0^2} \frac{\partial^2}{\partial \tau^2} \Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau), \tag{2.16}$$

with $j \in (1, 2)$, where ∇_j^2 is the Laplace operator operating to $\mathbf{r}_j$. Due to the Fourier-transform relation between Γ and W, it can be shown that the CSD function must satisfy the Helmholtz equations [52]

$$\nabla_j^2 W(\mathbf{r}_1, \mathbf{r}_2, \omega) + k^2 W(\mathbf{r}_1, \mathbf{r}_2, \omega) = 0, \tag{2.17}$$

with $j \in (1, 2)$. These equations are called the propagation law for the CSD function.

Let us next investigate the field emitted into the half-space $z > 0$ by a finite and planar secondary source with an area D, such as an illuminated aperture in an opaque screen. Based on Eq. (2.17), the CSD function of such a field in the far zone can be written in terms

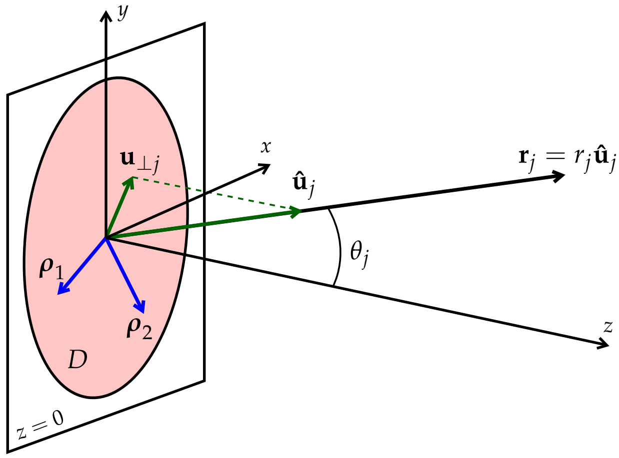

Figure 2.2: Geometry and notations for the propagation analysis. A planar source of area D in the plane $z = 0$ radiates into the half-space $z > 0$. The field correlations at far-zone positions $\mathbf{r}_j = r_j\hat{\mathbf{u}}_j$ are considered.

of the CSD function in the source plane $z = 0$ as [52, 61, 62]

$$W^{(\infty)}(r_1\hat{\mathbf{u}}_1, r_2\hat{\mathbf{u}}_2, \omega) = \left(\frac{k}{2\pi}\right)^2 \cos\theta_1 \cos\theta_2 \frac{e^{ik(r_2-r_1)}}{r_1 r_2} \times \iint_D W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) e^{-ik(\mathbf{u}_{\perp 2}\cdot\boldsymbol{\rho}_2 - \mathbf{u}_{\perp 1}\cdot\boldsymbol{\rho}_1)}\, \mathrm{d}^2\rho_1\, \mathrm{d}^2\rho_2, \tag{2.18}$$

where the far-zone positions $r_j\hat{\mathbf{u}}_j$ are located at the distances r_j from the source in the directions of $\hat{\mathbf{u}}_j$. Moreover, θ_j are the angles between $\hat{\mathbf{u}}_j$ and the z axis, $\boldsymbol{\rho}_j = (x_j, y_j)$ are positions in the source plane, and $\mathbf{u}_{\perp j} = (u_{xj}, u_{yj})$ are the transverse parts of $\hat{\mathbf{u}}_j$. Superscripts (0) and (∞) are used to emphasize that functions are evaluated at the source plane or in the far zone, respectively.

At frequency ω, the angular distribution of power flow, i.e., the rate at which the source radiates energy per unit solid angle around the direction specified by $\hat{\mathbf{u}}$, is given by [52, 61, 63]

$$\mathcal{J}(\hat{\mathbf{u}}, \omega) = \lim_{r\to\infty}\left[r^2 W^{(\infty)}(r\hat{\mathbf{u}}, r\hat{\mathbf{u}}, \omega)\right]. \tag{2.19}$$

The quantity $\mathcal{J}(\hat{\mathbf{u}}, \omega)$ is often used in radiometry and it is called the (spectral) radiant intensity. Further, the normalized spectral density of the far field can be expressed as the radiant intensity divided by the total radiant intensity,

$$s^{(\infty)}(\hat{\mathbf{u}}, \omega) = \frac{S^{(\infty)}(\hat{\mathbf{u}}, \omega)}{\int_0^\infty S^{(\infty)}(\hat{\mathbf{u}}, \omega)\,\mathrm{d}\omega} = \frac{\mathcal{J}(\hat{\mathbf{u}}, \omega)}{\int_0^\infty \mathcal{J}(\hat{\mathbf{u}}, \omega)\,\mathrm{d}\omega}. \tag{2.20}$$

The far-zone properties of other quantities of interest such as the spectral degree of coherence can be studied in a similar manner [52].

3 Electromagnetic theory of coherence

In scalar coherence theory an optical field is described with a scalar function. Thus, the analysis is generally restricted to paraxial, i.e., well-directional, and completely polarized fields. To fully understand the properties of light, however, it has to be expressed in terms of the electric and magnetic field vectors $\mathbf{E}_{\mathrm{re}}$ and $\mathbf{H}_{\mathrm{re}}$, respectively. These fields obey the fundamental Maxwell equations [4]. At optical frequencies the electric field is the main quantity as there usually is no direct interaction between the magnetic field and the matter. Thus, for the rest of the thesis we neglect the magnetic field in our analysis and concentrate on the electric field.

Interest towards the electromagnetic coherence theory has been increasing with the development of subwavelength nanostructures. Such structures give rise to near-field coherence phenomena, e.g., localized surface plasmons, that the scalar coherence theory is generally unable to model rigorously.

In this chapter the scalar theory discussed in Chapter 2 is extended to electromagnetic vector fields. The chapter begins with an introduction to the polarization of fields with two or three field components. Suitable matrices to describe the correlation phenomena of vector fields are presented, and the propagation in free space is addressed. Lastly, the degree of coherence for electromagnetic fields [37] is introduced, and the controversies related to the measures of electromagnetic coherence are discussed.

3.1 POLARIZATION

Polarization has a wide range of modern applications from sunglasses and liquid crystal displays to signal transmission and 3D movies, and it is the most notable difference between scalar and electromagnetic coherence theories. Polarization is a property of an electromagnetic field that describes how the tip of the electric field vector behaves as a function of time. The polarization properties of a field can be expressed with the help of two quantities, the degree of polarization and the polarization state of field's fully polarized part. The degree of polarization tells how big a portion of the field is fully polarized and the polarization state describes the movement pattern of the vector tip. Both of them are discussed in more detail in the following sections. The concept of polarization can also be extended to the case of non-paraxial fields with three orthogonal field components, as presented in Section 3.1.3.

3.1.1 Polarization matrix for two-component fields

One realization of a vector-valued electric field at a space–time point $(\mathbf{r}, t)$ is generally expressed as a three-component column vector

$$\mathbf{E}(\mathbf{r}, t) = \left[E_x(\mathbf{r}, t), E_y(\mathbf{r}, t), E_z(\mathbf{r}, t)\right]^{\mathrm{T}}, \tag{3.1}$$

where the complex analytic signal representation is employed analogously to the scalar case. Here the superscript T denotes the transpose and E_α, with $\alpha \in (x, y, z)$, are the Cartesian field components. Often, however, the field can be considered as a stationary paraxial beam propagating along the z axis which results in the electric field vibrating only in the xy plane. Thus, E_z can be considered to be zero and the polarization properties of the field are included in the 2×2 polarization matrix [29,33]

$$\mathbf{J}(\mathbf{r}) = \left\langle \mathbf{E}^*(\mathbf{r}, t)\mathbf{E}^{\mathrm{T}}(\mathbf{r}, t) \right\rangle. \tag{3.2}$$

The average intensities of the field components E_x and E_y are given by the diagonal elements of $\mathbf{J}$ whereas the off-diagonal elements represent the correlation between these components. The polarization matrix is Hermitian and non-negative definite [29,52].

An alternative method to describe the polarization of an electric field are the Stokes parameters, first introduced in 1852 [28]. The averaged Stokes parameters are defined using the elements of the polarization matrix, $J_{\alpha\beta}$, with $\alpha, \beta \in (x, y)$, as [29]

$$S_0(\mathbf{r}) = J_{xx}(\mathbf{r}) + J_{yy}(\mathbf{r}), \tag{3.3a}$$

$$S_1(\mathbf{r}) = J_{xx}(\mathbf{r}) - J_{yy}(\mathbf{r}), \tag{3.3b}$$

$$S_2(\mathbf{r}) = J_{yx}(\mathbf{r}) + J_{xy}(\mathbf{r}), \tag{3.3c}$$

$$S_3(\mathbf{r}) = \mathrm{i}\left[J_{yx}(\mathbf{r}) - J_{xy}(\mathbf{r})\right]. \tag{3.3d}$$

Physically, the quantity S_0 gives the total intensity of the field while the other three Stokes parameters deal with the intensity differences of various polarization states [64]. The parameter S_1 is the intensity difference between x- and y-polarized parts, S_2 gives the difference between $+45°$ and $-45°$ linearly polarized components, and S_3 can be interpreted as the difference between the right- and left-hand circularly polarized parts. The linear or circular polarization may be illustrated for fully coherent fields so that the tip of the electric field vector draws a line or a circle, respectively, when the field is evaluated at one point as a function of time.

Both the polarization matrix and the Stokes parameters can be represented also in the space–frequency domain analogously to Eqs. (3.2) and (3.3). As opposed to the time domain, such quantities describe the polarization properties of the electric field at a single frequency. Differences between the polarization in the time and frequency domains are discussed in [65, 66]. In the following sections the space–frequency representation is used due to its usefulness in the analysis of broadband light.

3.1.2 Degree of polarization

An electric field may be called fully polarized, unpolarized, or partially polarized based on how the field components are correlated. In the case of fully polarized light the x and y components of the electric field vector are completely correlated, i.e., $|j_{xy}(\mathbf{r}, \omega)| = 1$, where

$$j_{\alpha\beta}(\mathbf{r}, \omega) = \frac{J_{\alpha\beta}(\mathbf{r}, \omega)}{\left[J_{\alpha\alpha}(\mathbf{r}, \omega) J_{\beta\beta}(\mathbf{r}, \omega)\right]^{1/2}}, \tag{3.4}$$

with $(\alpha, \beta) \in (x, y)$, are the normalized elements of the polarization matrix $\mathbf{J}(\mathbf{r}, \omega)$. On the other hand, if the x and y components are completely uncorrelated, i.e., $|j_{xy}(\mathbf{r}, \omega)| = 0$, and their intensities are equal, the polarization matrix is proportional to the unit matrix and the field is said to be unpolarized. Partial polarization covers all the situations falling between these two extreme cases.

Due to the Hermiticity and the non-negative definiteness of $\mathbf{J}$, every polarization matrix has a unique decomposition as a sum of matrices corresponding to fully polarized and unpolarized light [29, 52]. This property gives rise to the possibility to describe a measure for the amount of polarization as one can separate the fully polarized portion from the total field. Thus, the degree of polarization is defined as the ratio between the spectral densities (or intensities) of the polarized part and the total field, and can be expressed in the equivalent forms [1,29,52]

$$P = \frac{\operatorname{tr} \mathbf{J}_\mathrm{p}}{\operatorname{tr} \mathbf{J}} = \left(1 - \frac{4 \det \mathbf{J}}{\operatorname{tr}^2 \mathbf{J}}\right)^{1/2} = \left[2\left(\frac{\operatorname{tr} \mathbf{J}^2}{\operatorname{tr}^2 \mathbf{J}} - \frac{1}{2}\right)\right]^{1/2}. \tag{3.5}$$

Here $\mathbf{J}_\mathrm{p}$ is the polarization matrix of the fully polarized part of the field, tr and det denote the trace and the determinant, respectively, and the dependence on position and frequency is omitted for brevity. The degree of polarization may obtain values between zero and unity, where $P = 0$ corresponds to an unpolarized field and $P = 1$ is attained when $\det \mathbf{J} = 0$ and the field is fully polarized.

3.1.3 Polarization of three-component fields

As noted earlier, it is possible to extend the concept of polarization to cover also near fields and other non-paraxial fields with three orthogonal field components [30, 31, 67]. In that case the polarization matrix is defined analogously to Eq. (3.2) as a 3×3 matrix $\mathbf{J}_3$, where the subscript 3 is used to emphasize that the electric field has now three orthogonal field components. In contrast to the case of paraxial fields, it is not possible to decompose $\mathbf{J}_3$ into fully polarized and unpolarized parts. However, based on the fact that polarization characterizes the correlations between the field components, the degree of polarization for three-component fields can be written as [31]

$$P_3 = \left[\frac{3}{2}\left(\frac{\mathrm{tr}\,\mathbf{J}_3^2}{\mathrm{tr}^2\,\mathbf{J}_3} - \frac{1}{3}\right)\right]^{1/2}. \tag{3.6}$$

Similarly to the degree of polarization in Eq. (3.5), P_3 is bounded by zero and unity where the former corresponds to unpolarized and the latter to fully polarized light.

Even though Eq. (3.6) looks similar to Eq. (3.5), when compared to the degree of polarization for two-component fields it is noticed that P_3 does not reduce to P when the field is well-directional, i.e., it contains only two orthogonal field components. This is intuitively clear as vibrations of any electric field with only two components are not completely random in three-dimensional space. However, such a property has been seen to be problematic, e.g., by Ellis *et al.* [68], who have proposed a degree of polarization that equals P in the case of two-component fields. Other suggestions have been given for example by Luis [69], Réfrégier *et al.* [70], and Dennis [71]. Moreover, the properties of these degrees have been studied by Sheppard [72], and Gamel and James [73], among others. For a more thorough discussion on the subject, see for example [32, 74].

3.2 COHERENCE OF ELECTROMAGNETIC FIELDS

While the polarization of vector-valued fields deals with the correlations between orthogonal field components at a single point, the coherence extends the analysis to cover similar correlations also between two positions. For stationary fields the space–time domain correlations are described by the mutual coherence matrix [33, 52]

$$\mathbf{\Gamma}(\mathbf{r}_1, \mathbf{r}_2, \tau) = \left\langle \mathbf{E}^*(\mathbf{r}_1, t)\mathbf{E}^{\mathrm{T}}(\mathbf{r}_2, t+\tau) \right\rangle, \tag{3.7}$$

where the electric field vector $\mathbf{E}$ may have two or three components depending on the situation. By comparing Eqs. (3.2) and (3.7) it is clear that the polarization matrix is equal to the mutual coherence matrix evaluated at a single space–time point, i.e., $\mathbf{J}(\mathbf{r}) = \mathbf{\Gamma}(\mathbf{r}, \mathbf{r}, 0)$. Moreover, the average intensity of an electromagnetic field is given by $I_{\mathrm{em}}(\mathbf{r}) = \mathrm{tr}\,\mathbf{\Gamma}(\mathbf{r}, \mathbf{r}, 0)$, where the subscript "em" is used to separate an electromagnetic quantity from its scalar counterpart.

Analogously to the scalar coherence theory, the electric CSD matrix $\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, \omega)$ can be defined through the Fourier-transform relations [52,75]

$$\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{\Gamma}(\mathbf{r}_1, \mathbf{r}_2, \tau) e^{\mathrm{i}\omega\tau}\, \mathrm{d}\tau, \tag{3.8a}$$

$$\mathbf{\Gamma}(\mathbf{r}_1, \mathbf{r}_2, \tau) = \int_{0}^{\infty} \mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, \omega) e^{-\mathrm{i}\omega\tau}\, \mathrm{d}\omega, \tag{3.8b}$$

that are the vector form of the generalized Wiener–Khintchine theorem for stationary fields. The matrix-functions $\mathbf{\Gamma}$ and $\mathbf{W}$ are Hermitian and non-negative definite in the sense discussed in [52]. The CSD matrix also has an expression as a correlation matrix [75,76]

$$\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \left\langle \boldsymbol{\mathcal{E}}^*(\mathbf{r}_1, \omega) \boldsymbol{\mathcal{E}}^{\mathrm{T}}(\mathbf{r}_2, \omega) \right\rangle, \tag{3.9}$$

where $\boldsymbol{\mathcal{E}}(\mathbf{r}, \omega)$ is a suitable vector-valued random function.

3.3 PROPAGATION OF ELECTROMAGNETIC FIELDS IN FREE SPACE

In this section the radiation from a similar source as in Section 2.6, i.e., a finite and planar secondary source in the plane $z = 0$ that emits light into the half-space $z > 0$, is considered (see Fig. 2.2). Analogously to the scalar case, it is possible to obtain a relation between CSD matrices in the source plane and the far zone. This can be achieved by using the angular correlation matrix

$$\mathbf{T}(k\mathbf{u}_{\perp 1}, k\mathbf{u}_{\perp 2}, \omega) = \frac{1}{(2\pi)^4} \iint_D \mathbf{W}^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) \times e^{-ik(\mathbf{u}_{\perp 2}\cdot\boldsymbol{\rho}_2 - \mathbf{u}_{\perp 1}\cdot\boldsymbol{\rho}_1)}\, \mathrm{d}^2\rho_1\, \mathrm{d}^2\rho_2, \tag{3.10}$$

which describes the cross-correlations between the plane-wave components of the angular spectrum. Here the integration is performed over the source area D. The CSD matrix in the far zone can be written as [77–79]

$$\mathbf{W}^{(\infty)}(r_1\hat{\mathbf{u}}_1, r_2\hat{\mathbf{u}}_2, \omega) = (2\pi k)^2 u_{z1} u_{z2} \frac{e^{ik(r_2-r_1)}}{r_1 r_2} \mathbf{T}(k\mathbf{u}_{\perp 1}, k\mathbf{u}_{\perp 2}, \omega). \tag{3.11}$$

Further, the radiant intensity for vector fields is defined analogously to Eq. (2.19) with the exception of using the electromagnetic spectral density $S_{\mathrm{em}}(\mathbf{r}, \omega) = \mathrm{tr}\, \mathbf{W}(\mathbf{r}, \mathbf{r}, \omega)$ instead of the scalar one, i.e.,

$$\mathcal{J}_{\mathrm{em}}(\hat{\mathbf{u}}, \omega) = \lim_{r\to\infty} \left[r^2 S_{\mathrm{em}}^{(\infty)}(r\hat{\mathbf{u}}, \omega) \right] = (2\pi k u_z)^2 \,\mathrm{tr}\, \mathbf{T}(k\mathbf{u}_\perp, k\mathbf{u}_\perp, \omega). \tag{3.12}$$

The normalized spectrum of the far field is thus given by

$$s_{\mathrm{em}}^{(\infty)}(\hat{\mathbf{u}}, \omega) = \frac{S_{\mathrm{em}}^{(\infty)}(\hat{\mathbf{u}}, \omega)}{\int_0^\infty S_{\mathrm{em}}^{(\infty)}(\hat{\mathbf{u}}, \omega)\, \mathrm{d}\omega} = \frac{\mathcal{J}_{\mathrm{em}}(\hat{\mathbf{u}}, \omega)}{\int_0^\infty \mathcal{J}_{\mathrm{em}}(\hat{\mathbf{u}}, \omega)\, \mathrm{d}\omega}. \tag{3.13}$$

It should be pointed out that the above analysis is equally valid for electric fields with two or three field components.

3.4 DEGREE OF COHERENCE FOR ELECTROMAGNETIC FIELDS

In scalar coherence theory the correlations between fields manifest themselves as the intensity modulations on the observation screen in Young's two-pinhole experiment. With vector fields, however, the situation is more involved due to the need to take into account correlations between all the field components. Thus, a straightforward generalization of the intensity fringe visibility from the scalar case is not a satisfactory measure of coherence in the electromagnetic domain, as will be shown explicitly in Section 3.4.2.

Regardless, a scalar quantity expressing the amount of correlation between the vector-valued electric fields at two points in space was introduced by Tervo *et al.* in 2003 [37]. In the space–time domain this degree of coherence for electromagnetic fields is written as the Frobenius norm [80] of the mutual coherence matrix, i.e.,

$$\gamma_{\mathrm{em}}(\mathbf{r}_1, \mathbf{r}_2, \tau) = \left\{ \frac{\mathrm{tr}\, \left[\mathbf{\Gamma}(\mathbf{r}_1, \mathbf{r}_2, \tau) \mathbf{\Gamma}(\mathbf{r}_2, \mathbf{r}_1, -\tau) \right]}{I_{\mathrm{em}}(\mathbf{r}_1) I_{\mathrm{em}}(\mathbf{r}_2)} \right\}^{1/2}, \tag{3.14}$$

and in the space–frequency domain analogously as [81, 82]

$$\mu_{\text{em}}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \left\{ \frac{\text{tr}\,[\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, \omega)\mathbf{W}(\mathbf{r}_2, \mathbf{r}_1, \omega)]}{S_{\text{em}}(\mathbf{r}_1, \omega) S_{\text{em}}(\mathbf{r}_2, \omega)} \right\}^{1/2}. \tag{3.15}$$

Both quantities γ_{em} and μ_{em} are real and may obtain values from zero to unity. If the degree of coherence for electromagnetic fields equals zero there are no correlations between any field components at positions $\mathbf{r}_1$ and $\mathbf{r}_2$, while unity corresponds to complete correlation between all field components.

These degrees have several properties that make them suitable measures of electromagnetic coherence. They remain invariant in unitary transformations which is a necessary feature for a good physical measure. Moreover, if the investigated field can be represented with a single field component, they reduce to the scalar degrees of coherence [37]. Thus, the degrees in Eqs. (3.14) and (3.15) are valid for fields with one, two, or three field components. They are also consistent with Glauber's definition of complete coherence [43], i.e., the corresponding degree of coherence equals unity only when the CSD matrix or the mutual coherence matrix is of a factored form [81, 83]. For example, if a field is completely coherent in the space–frequency domain, its CSD matrix is of the form [81]

$$\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \boldsymbol{\mathcal{V}}^*(\mathbf{r}_1, \omega)\boldsymbol{\mathcal{V}}^{\text{T}}(\mathbf{r}_2, \omega), \tag{3.16}$$

where $\boldsymbol{\mathcal{V}}(\mathbf{r}, \omega)$ is a vector that depends on the spectral densities of individual field components. Similarly to the scalar theory of coherence, also the CSD matrix can be decomposed into coherent modes [75, 84].

Moreover, if the degrees are evaluated at a single space–time or space–frequency point, they reduce to functions of the degree of polarization [37]. This may sound illogical at first due to the self-coherence property of the scalar degree of coherence, but it is actually a brilliant illustration of the differences between electromagnetic and scalar coherence phenomena. In the case of vector-valued fields the orthogonal components of the electric field at a single position may not be mutually completely correlated, i.e., the field is not necessarily fully polarized. In such a situation there is no complete correlation between all the field components so the field is not completely coherent. Thus, polarization can be regarded as the self-coherence of electromagnetic fields.

3.4.1 Young's interference experiment with electromagnetic fields

As their scalar counterparts, the degrees of coherence for electromagnetic fields are closely related to Young's interference experiment. In this subsection the experiment detailed in Section 2.3 and Fig. 2.1 is analyzed in the spectral domain with the distinction of having vector-valued electric fields in the pinholes. The investigated field is assumed to be paraxial so the electric field is described by a two-component vector $\mathbf{E}(\mathbf{r},\omega) = \left[E_x(\mathbf{r},\omega), E_y(\mathbf{r},\omega)\right]^{\mathrm{T}}$.

Now the electric field at the observation screen $\mathcal{B}$ can be written as [85, 86]

$$\mathbf{E}(\mathbf{r},\omega) = L_1\mathbf{E}(\mathbf{Q}_1,\omega)e^{\mathrm{i}kR_1} + L_2\mathbf{E}(\mathbf{Q}_2,\omega)e^{\mathrm{i}kR_2}, \tag{3.17}$$

where L_1 and L_2 are purely imaginary constants that depend on the pinholes and geometry as K_j in the scalar case. Thus, the polarization matrix at the screen is given by

$$\begin{aligned}\mathbf{J}(\mathbf{r},\omega) = {} & |L_1|^2\mathbf{J}(\mathbf{Q}_1,\omega) + |L_2|^2\mathbf{J}(\mathbf{Q}_2,\omega) + L_1^*L_2 \\ & \times \left[\mathbf{W}(\mathbf{Q}_1,\mathbf{Q}_2,\omega)e^{\mathrm{i}k(R_2-R_1)} + \mathbf{W}(\mathbf{Q}_2,\mathbf{Q}_1,\omega)e^{-\mathrm{i}k(R_2-R_1)}\right],\end{aligned} \tag{3.18}$$

where the first two terms correspond to the polarization matrix at the screen if only one of the pinholes is open.

Next, to gain better physical insight into Eq. (3.18), the generalized (two-point) Stokes parameters are defined as [41, 42]

$$\mathcal{S}_0(\mathbf{r}_1,\mathbf{r}_2,\omega) = W_{xx}(\mathbf{r}_1,\mathbf{r}_2,\omega) + W_{yy}(\mathbf{r}_1,\mathbf{r}_2,\omega), \tag{3.19a}$$
$$\mathcal{S}_1(\mathbf{r}_1,\mathbf{r}_2,\omega) = W_{xx}(\mathbf{r}_1,\mathbf{r}_2,\omega) - W_{yy}(\mathbf{r}_1,\mathbf{r}_2,\omega), \tag{3.19b}$$
$$\mathcal{S}_2(\mathbf{r}_1,\mathbf{r}_2,\omega) = W_{yx}(\mathbf{r}_1,\mathbf{r}_2,\omega) + W_{xy}(\mathbf{r}_1,\mathbf{r}_2,\omega), \tag{3.19c}$$
$$\mathcal{S}_3(\mathbf{r}_1,\mathbf{r}_2,\omega) = \mathrm{i}\left[W_{yx}(\mathbf{r}_1,\mathbf{r}_2,\omega) - W_{xy}(\mathbf{r}_1,\mathbf{r}_2,\omega)\right]. \tag{3.19d}$$

These parameters are obviously two-point versions of the traditional Stokes parameters presented in Eqs. (3.3), so they deal with correlations at two positions. Much in analogy with the classic Stokes parameters, the two-point Stokes parameters can be shown to involve sums and differences of correlations of different field components between points $\mathbf{r}_1$ and $\mathbf{r}_2$ [87].

Based on Eq. (3.18) the traditional Stokes parameters at the observation screen in Young's two-pinhole experiment can now be expressed in the form [85]

$$\begin{aligned} S_n(\mathbf{r},\omega) = {} & |L_1|^2 S_n(\mathbf{Q}_1,\omega) + |L_2|^2 S_n(\mathbf{Q}_2,\omega) \\ & + 2L_1^* L_2 \left[S_0(\mathbf{Q}_1,\omega) S_0(\mathbf{Q}_2,\omega)\right]^{1/2} |\eta_n(\mathbf{Q}_1,\mathbf{Q}_2,\omega)| \\ & \times \cos\left[\alpha_n(\mathbf{Q}_1,\mathbf{Q}_2,\omega) + k(R_2 - R_1)\right], \end{aligned} \tag{3.20}$$

with $n \in (0,1,2,3)$, where $\alpha_n(\mathbf{Q}_1,\mathbf{Q}_2,\omega) = \arg\{\eta_n(\mathbf{Q}_1,\mathbf{Q}_2,\omega)\}$, and

$$\eta_n(\mathbf{r}_1,\mathbf{r}_2,\omega) = \frac{\mathcal{S}_n(\mathbf{r}_1,\mathbf{r}_2,\omega)}{\left[S_0(\mathbf{r}_1,\omega) S_0(\mathbf{r}_2,\omega)\right]^{1/2}} \tag{3.21}$$

are the generalized Stokes parameters normalized by spectral densities. In the spirit of Eq. (2.7), Eq. (3.20) is called the electromagnetic spectral interference law [85]. According to it, all the Stokes parameters are modulated at the screen. Hence, in contrast to the scalar experiment, in the electromagnetic theory the field correlations manifest themselves also in the form of polarization modulation. Analogously to the visibility of interference fringes in the scalar case, the modulations of these Stokes parameters are described by the contrast parameters $|\eta_n(\mathbf{Q}_1,\mathbf{Q}_2,\omega)|$ [82,85,88].

As the spectral degree of coherence for electromagnetic fields and the contrast parameters both describe the amount of correlation between electromagnetic fields at two positions, it is quite natural to have a relation between them. This relation, written as [82]

$$\mu_{\mathrm{em}}^2(\mathbf{Q}_1,\mathbf{Q}_2,\omega) = \frac{1}{2}\sum_{n=0}^{3} |\eta_n(\mathbf{Q}_1,\mathbf{Q}_2,\omega)|^2, \tag{3.22}$$

shows that the spectral degree of coherence for electromagnetic fields is related to the sum of the contrasts of modulation in Young's interference experiment. This is an extension of the scalar coherence theory. These contrast parameters, as well as $\mu_{\text{em}}(\mathbf{Q}_1, \mathbf{Q}_2, \omega)$, can be measured by applying polarizers and wave plates in Young's interferometer [82, 89].

3.4.2 Other propositions for electromagnetic degree of coherence

The definition of a suitable measure of electromagnetic coherence has generated plenty of debate in literature, especially during the past decade, and the degrees of coherence for electromagnetic fields given in Eqs. (3.14) and (3.15) are not the only measures that have been proposed. The straightforward extension of the scalar degree of coherence was first put forward by Karczewski in the 1960s using the time-domain representation [34, 35] and later, in 2003, by Wolf in the spectral domain [36]. They defined the degree as the visibility of the intensity fringe pattern in Young's two-pinhole experiment when intensities in the pinholes are equal. Such a measure corresponds to the normalized two-point Stokes parameter $\eta_0(\mathbf{Q}_1, \mathbf{Q}_2, \omega)$, meaning that it neglects the polarization properties of the field completely. Additionally, $\eta_0(\mathbf{Q}_1, \mathbf{Q}_2, \omega)$ changes in coordinate or other unitary transformations which limits its use as a physical measure. Moreover, it is not consistent with the factorization property in the case of completely coherent light.

In 2005 Réfrégier and Goudail introduced two intrinsic degrees of coherence as the singular values of the matrix [38, 90]

$$\mathbf{M}_{\text{R}}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \mathbf{W}^{-1/2}(\mathbf{r}_2, \mathbf{r}_2, \omega)\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, \omega)\mathbf{W}^{-1/2}(\mathbf{r}_1, \mathbf{r}_1, \omega). \tag{3.23}$$

Such degrees are invariant under unitary transformations and the greater of them expresses essentially the maximum correlation between any two field components at $\mathbf{r}_1$ and $\mathbf{r}_2$ [91]. However, also these degrees fail to meet the factorization requirement.

The next measure of electromagnetic coherence was suggested by Luis in 2007 [39, 92]. He defined the degree of coherence as the distance between the block matrix

$$\mathbf{M}_{\mathrm{L}}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \begin{bmatrix} \mathbf{W}(\mathbf{r}_1, \mathbf{r}_1, \omega) & \mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, \omega) \\ \mathbf{W}(\mathbf{r}_2, \mathbf{r}_1, \omega) & \mathbf{W}(\mathbf{r}_2, \mathbf{r}_2, \omega) \end{bmatrix}, \tag{3.24}$$

and the identity matrix as

$$D_{\mathrm{L}} = \frac{4}{3} \left[\frac{\operatorname{tr} \mathbf{M}_{\mathrm{L}}^2(\mathbf{r}_1, \mathbf{r}_2, \omega)}{\operatorname{tr}^2 \mathbf{M}_{\mathrm{L}}(\mathbf{r}_1, \mathbf{r}_2, \omega)} - \frac{1}{4} \right]. \tag{3.25}$$

Similarly to γ_{em} and μ_{em}, also the degree D_{L} stays invariant under unitary transformations and factors in the case of complete coherence [39].

The studies on the maximum fringe visibility in Young's interference experiment by Gori *et al.* [93] and Martínez-Herrero and Mejías [94] in 2007 are also closely related to the degrees of electromagnetic coherence. In those studies the fields in the pinholes were altered with unitary transformations to find the maximum value for the visibility at the observation screen. The result in the time domain was expressed as [93]

$$|V_{\mathrm{I}}|^2_{\mathrm{max}}(\mathbf{r}) = \gamma^2_{\mathrm{em}}(\mathbf{r}_1, \mathbf{r}_2, \tau) + 2|\det \boldsymbol{\Gamma}(\mathbf{r}_1, \mathbf{r}_2, \tau)|, \tag{3.26}$$

and an equivalent spectral formula was derived in [94].

In summary, it is reasonable to assume that a satisfactory measure of electromagnetic coherence fulfills at least the following two conditions. It must remain invariant in unitary transformations and meet Glauber's spatial factorization condition in the case of complete coherence. Hence, only the degrees γ_{em} and μ_{em}, and the quantity D_{L} can be considered as proper measures of coherence.

4 Purity of electromagnetic fields

In general, the normalized spectrum and the polarization state of electromagnetic fields change on propagation due to diffraction and partial coherence. This happens in Young's interference experiment [95–98] and even in free space [27, 99]. However, there exist certain conditions that, when fulfilled, result in the invariance of the spectrum or the polarization properties of the field on propagation or on interference. The spectral invariance on propagation is examined in Chapter 5 whereas the present chapter is dedicated to various invariance phenomena occurring in Young's experiment.

Purity is a notion employed in coherence theory when some field property remains unchanged in the interference in Young's two-pinhole experiment. For example, Mandel [18] defined cross-spectrally pure fields to be such that their normalized spectra stay invariant on two-beam interference, and Gori *et al.* [48] divided unpolarized beams to pure and impure classes depending on whether the interference pattern in Young's experiment remains unpolarized or not. In this chapter the definition of cross-spectral purity is first recalled and analyzed in the case of scalar fields. The concept is then extended to cover electromagnetic fields according to paper **I** and the Stokes parameters based on paper **II**. The chapter is concluded with the analysis of polarization-invariance in Young's interference experiment according to paper **III**.

4.1 DEFINITION OF CROSS-SPECTRAL PURITY

Mandel's original motivation that led to the introduction of cross-spectral purity was the reduction property of the complex degree of coherence arisen in earlier studies [18]. For example in the analysis of the Hanbury Brown–Twiss (HBT) experiment γ was considered to factor into parts representing either spatial or temporal coherence. Mandel approached the situation by considering Young's two-pinhole experiment with the same normalized spectra in the pinholes. He said that the field is cross-spectrally pure if there exists a point in the observation plane in whose neighborhood the normalized spectrum coincides with that in the pinholes. Such a situation is illustrated in Fig. 4.1. This kind of definition leads to the reduction property as will be shown next.

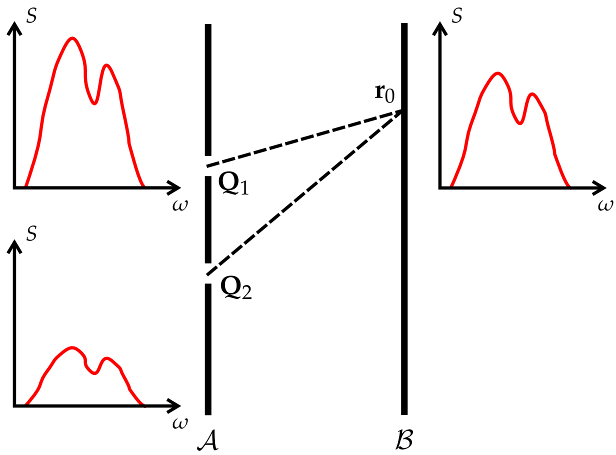

Figure 4.1: Illustration of cross-spectrally pure light in Young's interference experiment. The normalized spectra in the pinholes Q_1 and Q_2 are equal, and at the observation screen there is a point $\mathbf{r}_0$ where the normalized spectrum coincides with that.

Let us consider the spectral densities in the pinholes to be proportional to each other, i.e., $S(\mathbf{Q}_2, \omega) = C_{12} S(\mathbf{Q}_1, \omega)$ for all ω, where C_{12} is a real and positive scalar number. Now, based on the spectral version of the interference law in Eq. (2.7), the spectrum at the observation screen is of the form [19, 52]

$$S(\mathbf{r}, \omega) = S(\mathbf{Q}_1, \omega)\Big[|K_1|^2 + C_{12}|K_2|^2 + 2\sqrt{C_{12}}|K_1||K_2| \times \Re\left\{\mu(\mathbf{Q}_1, \mathbf{Q}_2, \omega) e^{-\mathrm{i}\omega\tau}\right\}\Big], \tag{4.1}$$

where $\Re$ denotes the real part and τ is again the time difference that occurs when light travels from the pinholes to $\mathbf{r}$. For the field to meet the requirements of cross-spectral purity there has to be some point $\mathbf{r}_0$ in the observation plane where the spectral density is proportional to $S(\mathbf{Q}_1, \omega)$ for all frequencies. This condition is obviously satisfied if the term in the square brackets in Eq. (4.1) is independent of frequency for some τ_0 corresponding to the position $\mathbf{r}_0$. As the frequency-dependence of the terms K_j can be neglected if the light in the pinholes has a narrow enough spectrum [52], the requirement needed to achieve cross-spectral purity can be written as

$$\mu(\mathbf{Q}_1, \mathbf{Q}_2, \omega) e^{-\mathrm{i}\omega\tau_0} = f(\mathbf{Q}_1, \mathbf{Q}_2, \tau_0), \tag{4.2}$$

where $f(\mathbf{Q}_1, \mathbf{Q}_2, \tau_0)$ is some arbitrary frequency-independent function.

To gain physical insight into the role of f, the definitions of the spectral degree of coherence in Eq. (2.13), the CSD function in Eq. (2.11), and the complex degree of coherence in Eq. (2.4) are recalled. Combining them with Eq. (4.2) yields [52]

$$\gamma(\mathbf{Q}_1, \mathbf{Q}_2, \tau) = f(\mathbf{Q}_1, \mathbf{Q}_2, \tau_0)\gamma(\mathbf{Q}_1, \mathbf{Q}_1, \tau - \tau_0). \tag{4.3}$$

As $\gamma(\mathbf{Q}_1, \mathbf{Q}_1, 0) = 1$, by setting $\tau = \tau_0$ we see that $f(\mathbf{Q}_1, \mathbf{Q}_2, \tau_0) = \gamma(\mathbf{Q}_1, \mathbf{Q}_2, \tau_0)$ and obtain the reduction formula for cross-spectrally pure fields in the form [18]

$$\gamma(\mathbf{Q}_1, \mathbf{Q}_2, \tau) = \gamma(\mathbf{Q}_1, \mathbf{Q}_2, \tau_0)\gamma(\mathbf{Q}_1, \mathbf{Q}_1, \tau - \tau_0). \tag{4.4}$$

According to it the complex degree of coherence of cross-spectrally pure fields factors into two parts, one characterizing spatial and the other temporal coherence. Moreover, inserting f into Eq. (4.2) yields another formula with a clear physical meaning [19]:

$$\mu(\mathbf{Q}_1, \mathbf{Q}_2, \omega) e^{-\mathrm{i}\omega\tau_0} = \gamma(\mathbf{Q}_1, \mathbf{Q}_2, \tau_0). \tag{4.5}$$

It states that the absolute value of the spectral degree of coherence is the same at every frequency and equal to the degree of coherence at time difference τ_0. Mandel also introduced the concept of the degree of cross-spectral purity as a measure of how cross-spectrally pure a field is [18].

Later, cross-spectral purity was experimentally demonstrated by Kandpal *et al.* [100], and the concept was also investigated by James and Wolf [101] who examined a situation of cross-spectral purity in some volume. Their analysis was based on the assumptions that, inside the volume, the normalized spectrum is the same at each point and the spectral degree of coherence is of the cross-spectrally pure form at each pair of points. As one example they showed that the far-field generated by a planar and quasi-homogeneous secondary source, examined earlier in Section 2.6, is generally not cross-spectrally pure in a volume. However, if the source obeys a so-called scaling law [27, 102], discussed more in Chapter 5, the far-field fulfills the condition of having the same normalized spectrum at every point. Further, the field can be considered cross-spectrally pure if the source is also quasi-monochromatic. Hence, the cross-spectral purity of the far-field emitted by a planar and quasi-homogeneous source obeying the scaling law was concluded to justify the use of the reduction formula in the analysis of the HBT experiment [101]. This was also one of Mandel's original objects of interest.

4.2 CROSS-SPECTRAL PURITY OF ELECTROMAGNETIC FIELDS

All the investigations of cross-spectral purity mentioned in the previous section have been performed using the scalar coherence theory. This restricts the analysis to a scalar, or uniformly and fully polarized field. The concept is extended to paraxial electromagnetic fields with two field components in paper **I** in a manner briefly outlined in this section.

As in the scalar case, we examine Young's interference experiment in electromagnetic context in a situation where the normalized spectral densities in the pinholes are equal and there exists a point in the observation plane with the same normalized spectral density. Employing the electromagnetic spectral interference law and the generalized Wiener–Khintchine theorem, Eqs. (3.20) and (3.8), leads to conditions of cross-spectral purity that are analogous to Eqs. (4.4) and (4.5) in the scalar case. The first condition, or the reduction formula for cross-spectrally pure electromagnetic fields, states that the normalized temporal two-point Stokes parameter $\nu_0(\mathbf{Q}_1, \mathbf{Q}_2, \tau)$ factors into spatial and temporal parts as

$$\nu_0(\mathbf{Q}_1, \mathbf{Q}_2, \tau) = \nu_0(\mathbf{Q}_1, \mathbf{Q}_2, \tau_0)\nu_0(\mathbf{Q}_1, \mathbf{Q}_1, \tau - \tau_0), \tag{4.6}$$

where

$$\nu_0(\mathbf{Q}_1, \mathbf{Q}_2, \tau) = \frac{\mathrm{tr}\,\Gamma(\mathbf{Q}_1, \mathbf{Q}_2, \tau)}{[I_{\mathrm{em}}(\mathbf{Q}_1)I_{\mathrm{em}}(\mathbf{Q}_2)]^{1/2}}. \tag{4.7}$$

Further, according to the second condition

$$\eta_0(\mathbf{Q}_1, \mathbf{Q}_2, \omega)e^{-\mathrm{i}\omega\tau_0} = \nu_0(\mathbf{Q}_1, \mathbf{Q}_2, \tau_0), \tag{4.8}$$

the absolute value of the normalized spectral two-point Stokes parameter $\eta_0(\mathbf{Q}_1, \mathbf{Q}_2, \omega)$ is independent of frequency and equal to $|\nu_0(\mathbf{Q}_1, \mathbf{Q}_2, \tau_0)|$.

Despite the similarity of the mathematical results in scalar and electromagnetic cases, there are differences between the physical interpretations. For example, the quantities ν_0 and η_0 are related to the fringe visibility in Young's experiment like γ and μ in the scalar theory, but their role as measures of electromagnetic coherence is rather ambiguous as discussed in Section 3.4.2. Moreover, an electromagnetic field can be cross-spectrally pure even if the individual field components are not. This arises from the definition of the spectral density for vector fields. Respectively, it is possible for a superposition of x- and y-polarized cross-spectrally pure fields to be impure.

4.3 CROSS-SPECTRAL PURITY OF STOKES PARAMETERS

In the study of the electromagnetic cross-spectral purity the quantity S_0 representing the spectral density for vector fields is concerned. It is thus natural to wonder if a similar analysis could also be done for the other Stokes parameters expressing polarization properties. Such an investigation is performed in paper **II** as explained below.

Now, instead of the normalized spectral density, we take one of the normalized Stokes parameters to be equal in the pinholes. Hence, $S_n(\mathbf{Q}_2, \omega) = C_n S_n(\mathbf{Q}_1, \omega)$, where C_n is a proportionality constant, is supposed to hold for one $n \in (0, 1, 2, 3)$ at every frequency. Further, we consider a point $\mathbf{r}_n$ in the observation screen in whose neighborhood the same normalized Stokes parameter is equal to that in the pinholes as a function of ω. After a process analogous to the one in paper **I**, conditions for the cross-spectral purity of the Stokes parameters are obtained as follows:

$$\psi_n(\mathbf{Q}_1, \mathbf{Q}_2, \tau) = \psi_n(\mathbf{Q}_1, \mathbf{Q}_2, \tau_n)\psi_n(\mathbf{Q}_1, \mathbf{Q}_1, \tau - \tau_n), \tag{4.9}$$

and

$$\mu_n(\mathbf{Q}_1, \mathbf{Q}_2, \omega)e^{-i\omega\tau_n} = \psi_n(\mathbf{Q}_1, \mathbf{Q}_2, \tau_n). \tag{4.10}$$

Here μ_n is the spectral two-point Stokes parameters normalized by the corresponding Stokes parameters, i.e.,

$$\mu_n(\mathbf{Q}_1, \mathbf{Q}_2, \omega) = \frac{\mathcal{S}_n(\mathbf{Q}_1, \mathbf{Q}_2, \omega)}{[S_n(\mathbf{Q}_1, \omega)S_n(\mathbf{Q}_2, \omega)]^{1/2}}, \tag{4.11}$$

and ψ_n are their counterparts in the space–time domain.

It should be noted that the quantities ν_0 and η_0 used in the analysis of electromagnetic cross-spectral purity correspond to ψ_0 and μ_0, respectively. For other parameters the normalization method is different. The results (4.9) and (4.10) are similar to Eqs. (4.6) and (4.8) of the electromagnetic cross-spectral purity and can be interpreted physically in an analogous way.

Moreover, if all four Stokes parameters are cross-spectrally pure regarding a single position $\mathbf{r}_0$ in the observation plane and the polarization state is the same in the pinholes and around $\mathbf{r}_0$, the temporal and spectral degrees of coherence for electromagnetic fields at the pinholes are equal, i.e., $\gamma_{\text{em}} = \mu_{\text{em}}$. Hence, this kind of a situation of strict cross-spectral purity is related to the scalar cross-spectral purity in the sense that the temporal and spectral measures of coherence are equal.

4.4 POLARIZATION PURITY

The purity of polarization properties has been examined from a different point of view by Gori *et al.* [48], who investigated interference patterns of unpolarized beams in Young's experiment. The beams were called pure if the light remained unpolarized also at the observation screen and impure if the experiment broke the polarization state. Such polarization invariance was also studied earlier by Santarsiero without restricting the analysis to unpolarized light [103]. Santarsiero's results are reassessed in paper **III** together with a discussion on the differences between the space–time and space–frequency domains. Polarization purity deals with the polarization properties on superposition at a single frequency over the whole observation plane. This is the main difference as compared to the cross-spectral purity analyses in which the spectral density or one of the Stokes parameters is investigated over the whole frequency band in a small area at the observation plane. Such a distinction leads to interesting physical results as will be shown next.

The starting point for the analysis is again Young's two-pinhole experiment. This time the polarization matrices in the pinholes are taken to be proportional to each other, i.e.,

$$\mathbf{J}(\mathbf{Q}_j, \omega) = \beta_j(\omega)\mathbf{j}(\omega), \tag{4.12}$$

with $j \in (1, 2)$, where $\beta_j(\omega)$ are scalar intensity factors and $\mathbf{j}(\omega)$ is a normalized polarization matrix with $\mathrm{tr}\,\mathbf{j}(\omega) = 1$. The conditions for which the polarization matrix on the screen is proportional to $\mathbf{j}(\omega)$ are investigated, and by employing Eq. (3.18) it can be shown that the CSD matrix at the pinholes has to be expressable as

$$\mathbf{W}(\mathbf{Q}_1, \mathbf{Q}_2, \omega) = F(\mathbf{Q}_1, \mathbf{Q}_2, \omega)\mathbf{j}(\omega). \tag{4.13}$$

In other words, according to this condition of spectral polarization purity, the beams whose polarization is pure are characterized by the decoupling of the spatial correlation properties and the polarization state, defined by $F(\mathbf{Q}_1, \mathbf{Q}_2, \omega)$ and $\mathbf{j}(\omega)$, respectively. By comparison, in the case of cross-spectrally pure light the temporal and spatial correlation properties are decoupled.

To emphasize the general differences in polarization between the space–time and space–frequency domains [65, 104], it can further be shown that a field with pure polarization in the frequency domain is not necessarily pure in the time domain. On the other hand, time-domain purity leads to purity also in the frequency domain.

Polarization purity is also closely related to the non-quantum entanglement of polarization and spatial modulation considered, for example, by Simon *et al.* [105] as well as Qian and Eberly [106]. Generally, for the two-component electromagnetic fields of the form specified in Eq. (3.1) the polarization properties are inseparably entangled with the spatial variation. If the field components are proportional to each other, however, the field can be expressed in the form

$$\mathbf{E}(\mathbf{r}) = E_x(\mathbf{r}) \begin{bmatrix} 1 \\ a \end{bmatrix}, \tag{4.14}$$

where a is a proportionality constant and the dependence on time or frequency is omitted for brevity. For this kind of a field the polarization and the spatial modulation are separated. Such a separation leads to the purity of polarization as described in paper **III**.

5 Spectral invariance on propagation

In the previous chapter it was shown that the normalized spectrum and the polarization state of electromagnetic fields may change in Young's interference experiment. This is quite natural, as similar changes happen even on propagation in free space due to the partial spatial coherence of the source. The understanding of such phenomena has been an important part in demonstrating the differences between the near- and far-field spectra. It has also had an effect on astronomical spectroscopy where the properties of stars and other distant astronomical objects are interpreted from the spectral measurements performed very far from the source [107, 108]. The changes in the normalized spectrum on propagation were first studied by Wolf [27]. He derived conditions for the source that guarantee the spectral invariance of the emitted light in the far zone. Polarization invariance on propagation, on the other hand, was later studied by James [99], followed by others. In this chapter the analysis on scalar spectral invariance, originally performed by Wolf, is recalled and then extended to the case of electromagnetic fields with two or three field components based on paper **IV**.

5.1 SPECTRAL INVARIANCE OF SCALAR FIELDS

In his seminal article [27] Wolf analyzed a situation similar to the one examined in Section 2.6, i.e., the radiation into the half-space $z > 0$ from a finite and planar secondary source of area D, located in the plane $z = 0$ (for notations, see Fig. 2.2). Additionally, the source was considered to have a uniform spectral distribution and its dimensions were taken to be very large compared to the spectral correlation width. Moreover, the spectral degree of coherence was assumed to be of the statistically homogeneous form $\mu^{(0)}(\Delta\boldsymbol{\rho},\omega)$. This implies that the spectral correlations between two source points $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ depend only on the difference between the positions, $\Delta\boldsymbol{\rho} = \boldsymbol{\rho}_2 - \boldsymbol{\rho}_1$. Hence, the CSD function within the source area D can be expressed as

$$W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = S^{(0)}(\omega)\mu^{(0)}(\Delta\boldsymbol{\rho},\omega), \tag{5.1}$$

and the source is called quasi-homogeneous [61].

The normalized spectrum in the direction $\hat{\mathbf{u}}$ in the far zone is obtained by inserting Eq. (5.1) to Eq. (2.18) and evaluating Eq. (2.20). This yields

$$s^{(\infty)}(\hat{\mathbf{u}},\omega) = \frac{k^2 S^{(0)}(\omega)\tilde{\mu}^{(0)}(k\mathbf{u}_\perp,\omega)}{\int_0^\infty k^2 S^{(0)}(\omega)\tilde{\mu}^{(0)}(k\mathbf{u}_\perp,\omega)\,\mathrm{d}\omega}, \tag{5.2}$$

where

$$\tilde{\mu}^{(0)}(k\mathbf{u}_\perp,\omega) = \frac{1}{2\pi}\int \mu^{(0)}(\Delta\boldsymbol{\rho},\omega)e^{-ik\mathbf{u}_\perp\cdot\Delta\rho}\,\mathrm{d}^2(\Delta\rho) \tag{5.3}$$

is the two-dimensional spatial Fourier transform of $\mu^{(0)}$. From Eq. (5.2) it is evident that the normalized spectrum of the far field generated by a quasi-homogeneous source generally depends on direction. However, if $\tilde{\mu}^{(0)}$ can be factored into parts depending only on the frequency or the direction, i.e.,

$$\tilde{\mu}^{(0)}(k\mathbf{u}_\perp,\omega) = F(\omega)\tilde{H}(\mathbf{u}_\perp), \tag{5.4}$$

where F and $\tilde{H}$ are arbitrary functions, the directional dependence of the right-hand side of Eq. (5.2) vanishes. Furthermore, by taking the Fourier inverse of Eq. (5.4) and recalling that $\mu^{(0)}$ is a correlation function, it can be shown that F must be of the form $F(\omega) = \left[k^2 H(0)\right]^{-1}$, where H is the two-dimensional inverse Fourier transform of $\tilde{H}$. This leads to the normalized spectrum

$$s^{(\infty)}(\omega) = \frac{S^{(0)}(\omega)}{\int_0^\infty S^{(0)}(\omega)\,\mathrm{d}\omega}, \tag{5.5}$$

which clearly equals the normalized spectrum of the source.

The desired condition for spectral invariance on propagation into the far zone can now be obtained from Eq. (5.4) in the form

$$\mu^{(0)}(\Delta\boldsymbol{\rho}, \omega) = \frac{H(k\Delta\boldsymbol{\rho})}{H(0)}, \tag{5.6}$$

which Wolf called the scaling law [27, 102]. According to it, the normalized spectrum of the far field is independent of direction and equals the normalized spectrum of the source if the spectral degree of coherence of the source depends only on the quantity $k\Delta\boldsymbol{\rho}$. One class of sources fulfilling this condition are the quasi-homogeneous planar Lambertian sources, as their spectral degree of coherence is of the uniform form [21]

$$\mu^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = \frac{\sin(k|\Delta\boldsymbol{\rho}|)}{k|\Delta\boldsymbol{\rho}|}. \tag{5.7}$$

An example of such a secondary source is an opening in a black-body cavity [21].

5.2 SPECTRAL INVARIANCE OF ELECTROMAGNETIC FIELDS

The concept of spectral invariance on propagation into the far zone was later extended to paraxial electromagnetic fields by Pu *et al.* [109]. According to them the spectral invariance is achieved when the scalar scaling law (5.6) holds individually for both x and y components of the electric field. Such a condition, however, is only a special case of the scaling law for paraxial vector-valued fields as shown in paper **IV**. In that paper we derive an analog for Wolf's scalar scaling law for general electromagnetic fields with two or three field components. Moreover, we discuss its differences to the scaling law introduced in [109]. The results are briefly outlined in this section.

Just like in the scalar case, a planar and finite secondary source in the plane $z = 0$ is considered. The source radiates into the half-space $z > 0$ and the source correlations are taken to be statistically homogeneous. In the context of the electromagnetic coherence theory this implies that the CSD matrix within the source area D depends spatially only on $\Delta\boldsymbol{\rho}$. Such homogeneity leads to the spectral density being uniform across the source. Hence, the CSD matrix of the source can be expressed in the form

$$\mathbf{W}^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = S_{\mathrm{em}}^{(0)}(\omega)\mathbf{w}^{(0)}(\Delta\boldsymbol{\rho}, \omega), \tag{5.8}$$

where $\mathbf{w}^{(0)}$, defined as

$$\mathbf{w}^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = \frac{\mathbf{W}^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega)}{\left[S_{\mathrm{em}}^{(0)}(\boldsymbol{\rho}_1, \omega)S_{\mathrm{em}}^{(0)}(\boldsymbol{\rho}_2, \omega)\right]^{1/2}}, \tag{5.9}$$

is the intensity-normalized CSD matrix.

Following the scalar analysis, Eq. (5.8) is inserted into Eq. (3.10), after which Eqs. (3.12) and (3.13) yield the normalized spectrum of the far field as

$$s_{\mathrm{em}}^{(\infty)}(\hat{\mathbf{u}}, \omega) = \frac{k^2 S_{\mathrm{em}}^{(0)}(\omega)\mathrm{tr}\,\tilde{\mathbf{w}}^{(0)}(k\mathbf{u}_\perp, \omega)}{\int_0^\infty k^2 S_{\mathrm{em}}^{(0)}(\omega)\mathrm{tr}\,\tilde{\mathbf{w}}^{(0)}(k\mathbf{u}_\perp, \omega)\,\mathrm{d}\omega}, \tag{5.10}$$

where $\tilde{\mathbf{w}}^{(0)}$ is the two-dimensional spatial Fourier transform of $\mathbf{w}^{(0)}$. As in the scalar case, the normalized spectrum of the far field is generally direction-dependent. To fulfill the definition of spectral invariance on propagation in the electromagnetic case, a scaling law similar to Eq. (5.6) has to hold for the trace of the normalized CSD matrix, i.e.,

$$\mathrm{tr}\,\mathbf{w}^{(0)}(\Delta\boldsymbol{\rho}, \omega) = \frac{G(k\Delta\boldsymbol{\rho})}{G(0)}, \tag{5.11}$$

where G is an arbitrary function. This condition states that the normalized spectrum of the far field emitted by a planar, quasi-homogeneous source is direction-independent and equals the normalized spectrum of the source if $\mathrm{tr}\,\mathbf{w}^{(0)}$ depends only on the quantity $k\Delta\boldsymbol{\rho}$. Thus, it is called the scaling law for electromagnetic sources.

It should be noted that Eq. (5.11) is equally valid for fields with one, two, or three field components. Especially, in the case of paraxial fields, it is more general than the condition put forward in [109] as now it is not necessary for the individual field components to fulfill the scalar scaling law. This important difference between scalar and electromagnetic approaches is further discussed with examples in paper **IV**. In one example the individual field components do not obey the scaling law but, due to the spectral compensation between the components, the normalized far-field spectrum is still independent of direction and equals the normalized spectrum of the source. On the other hand, an example where the x and y components fulfill the scaling law individually but the normalized far-field spectrum still depends on direction is also constructed. Overall, the spectral invariance is a considerably more complex phenomenon in the case of vector-valued fields than with scalar fields.

6 Hanbury Brown–Twiss effect

In the previous chapters the classical second-order coherence theory was employed to analyze the correlations of field fluctuations in various invariance phenomena. The viewpoint of this chapter, however, is slightly different as now the coherence properties of light are investigated through the correlations of intensity fluctuations which manifest themselves as one class of classical fourth-order correlations. In the quantum theory of optical coherence, on the other hand, these correlations are said to be of second-order.

The correlations of intensity fluctuations are usually associated with the famous Hanbury Brown–Twiss (HBT) experiment that has had a significant impact on the development of astronomy and quantum physics [16, 17, 52]. In the original experiment the intensities of two light beams were measured with photomultiplier tubes and the correlations between these beams were analyzed via photoelectron current fluctuations. With suitable modifications such a system was used in intensity interferometry to measure, e.g., the angular diameters of distant stars. Later the effect has found important applications also in the fields of high-energy nuclear physics and atomic physics [110]. In this chapter the HBT experiment and its connection to coherence measurements are first discussed in detail in the context of scalar coherence theory. The extension of the HBT effect to cover also electromagnetic fields is then represented according to paper **V**.

6.1 HANBURY BROWN–TWISS EXPERIMENT

The setup of the HBT experiment basically consists of two movable detectors at positions $\mathbf{r}_1$ and $\mathbf{r}_2$, a possible delay line, and a correlator as illustrated in Fig. 6.1. Detectors measure the instantaneous intensities of light, $I_i(\mathbf{r}_j, t) = E^*(\mathbf{r}_j, t)E(\mathbf{r}_j, t)$, with $j \in (1, 2)$, that are converted to photoelectron currents $i(\mathbf{r}_j, t)$. These photocurrents are usually taken proportional to the intensities so that the measurable currents give information about intensities directly. The delay line can be employed to compensate for the different distances from source to detectors and to delay one signal by the desired time difference τ. Lastly, the signals are measured and correlated in the correlator. Hence, such a system can be used to evaluate both the spatial and the temporal correlations.

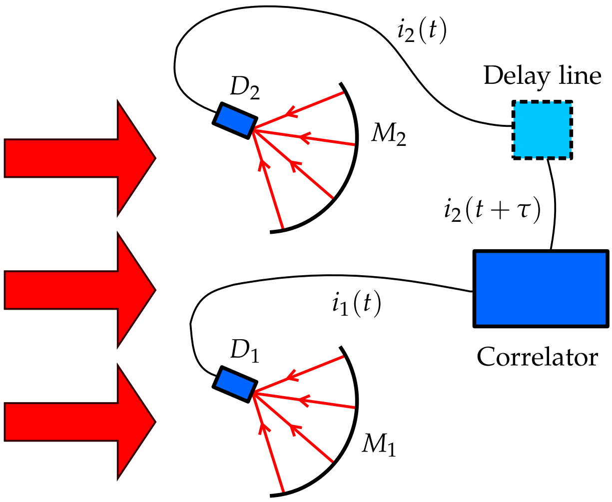

Figure 6.1: Setup for the HBT experiment. Incoming light is collected by the mirrors M_1 and M_2 and focused onto the detectors D_1 and D_2. The photocurrents i_1 and i_2 generated by the detectors are evaluated in the correlator. A delay line may be introduced to delay one of the photocurrents by a time difference τ.

As natural light is chaotic, its intensity fluctuates rapidly. These fluctuations, defined as the difference between the instantaneous intensity and the average intensity, $\Delta I(\mathbf{r}, t) = I_i(\mathbf{r}, t) - I(\mathbf{r})$, can be measured with the HBT setup as the fluctuations of the photoelectron currents. More importantly, with the correlator it is possible to measure the correlations between the fluctuations, i.e.,

$$\langle \Delta I(\mathbf{r}_1, t)\Delta I(\mathbf{r}_2, t+\tau)\rangle = \langle I_i(\mathbf{r}_1, t)I_i(\mathbf{r}_2, t+\tau)\rangle - I(\mathbf{r}_1)I(\mathbf{r}_2), \quad (6.1)$$

where the light is again considered to be stationary. Furthermore, if the light originates from a thermal source such as a star or a light bulb, it is known to obey Gaussian probability statistics [52]. According to the Gaussian moment theorem [52,111], such statistics lead to the fourth-order correlation term on the right-hand side of Eq. (6.1) being expressable in terms of the second-order correlation functions as

$$\langle I_{\mathrm{i}}(\mathbf{r}_1, t) I_{\mathrm{i}}(\mathbf{r}_2, t+\tau) \rangle = I(\mathbf{r}_1) I(\mathbf{r}_2) + |\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau)|^2. \tag{6.2}$$

Thus, by inserting Eq. (6.2) into Eq. (6.1) and normalizing both sides with average intensities, it is seen at once that measuring the normalized correlations of the intensity fluctuations from a thermal source in the HBT experiment yields

$$\frac{\langle \Delta I(\mathbf{r}_1, t) \Delta I(\mathbf{r}_2, t+\tau) \rangle}{I(\mathbf{r}_1) I(\mathbf{r}_2)} = |\gamma(\mathbf{r}_1, \mathbf{r}_2, \tau)|^2, \tag{6.3}$$

i.e., the square of the degree of coherence.

The obtained result is remarkable as, according to it, the HBT experiment provides another means to measure correlations besides Young's interference experiment or Michelson interferometer [7]. The main advantage of this kind of intensity interferometry over earlier methods is that there is no need to overlap light beams to form a sensitive interference pattern as only the photoelectron currents are brought together. This allows longer measurement times which reduces the problems caused by the shaking of the ground and the atmospheric turbulence. In addition, the requirements imposed onto the optical elements of the system are eased and larger separations between the detectors are allowed. It has to be kept in mind, though, that Eq. (6.3) is valid only for scalar fields obeying Gaussian statistics.

6.2 ELECTROMAGNETIC ANALYSIS OF HANBURY BROWN–TWISS EFFECT

The extension of the HBT effect to vector-valued fields was briefly examined by Tervo *et al.* in 2003 [37]. They analyzed the experiment in the space–time domain as was done in the previous section and showed that the normalized intensity-correlation function specified in Eq. (6.3) equals the square of the degree of coherence for electromagnetic fields, i.e., $\gamma_{\mathrm{em}}^2(\mathbf{r}_1, \mathbf{r}_2, \tau)$, if the field obeys Gaussian statistics. Such a result is completely consistent with the scalar analysis.

Later the intensity fluctuations were reassessed in the space–frequency domain with the introduction of the so-called degree of cross-polarization [49–51]. The degree, expressable in terms of the contrast parameters through

$$\mathcal{P}^2(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{\sum_{n=1}^{3} |\eta_n(\mathbf{r}_1, \mathbf{r}_2, \omega)|^2}{|\eta_0(\mathbf{r}_1, \mathbf{r}_2, \omega)|^2}, \tag{6.4}$$

concerns correlations between two positions and reduces to the degree of polarization when $\mathbf{r}_1 = \mathbf{r}_2$. Moreover, the physical result in papers [49–51] was that, unlike in the scalar case, the degree of coherence does not completely describe the intensity fluctuations but the degree of cross-polarization is also needed. This was due to applying the electromagnetic spectral degree of coherence defined by Wolf [36]. However, the direct relationship between the HBT effect and the degree of coherence for electromagnetic fields was completely neglected in the above-mentioned papers.

In paper **V** the HBT effect is analyzed in the space–frequency domain with vectorial fields of any state of coherence and polarization obeying Gaussian statistics. A derivation analogous to the scalar case leads to the result

$$\frac{\langle \Delta S_{\mathrm{em}}(\mathbf{r}_1, \omega) \Delta S_{\mathrm{em}}(\mathbf{r}_2, \omega) \rangle}{S_{\mathrm{em}}(\mathbf{r}_1, \omega) S_{\mathrm{em}}(\mathbf{r}_2, \omega)} = \mu_{\mathrm{em}}^2(\mathbf{r}_1, \mathbf{r}_2, \omega), \tag{6.5}$$

where the main mathematical change from the scalar analysis is the different definition of the fluctuations of intensity or spectral density. The electromagnetic spectral-density fluctuations are defined as

$$\Delta S_{\mathrm{em}}(\mathbf{r}, \omega) = S_{\mathrm{em,i}}(\mathbf{r}, \omega) - \mathrm{tr}\, \mathbf{W}(\mathbf{r}, \mathbf{r}, \omega), \tag{6.6}$$

where $S_{\mathrm{em,i}}(\mathbf{r}, \omega) = \mathrm{tr}\left[\boldsymbol{\mathcal{E}}^*(\mathbf{r}, \omega) \boldsymbol{\mathcal{E}}^{\mathrm{T}}(\mathbf{r}, \omega)\right]$. According to Eq. (6.5) the degree of coherence alone is enough to describe also electromagnetic intensity fluctuations which is a physically different result from the one before. This is due to the different definition of the electromagnetic degree of coherence. As the degree of coherence for electromagnetic fields can be viewed as a sum of the contrast parameters in Young's interference experiment according to Eq. (3.22), the electromagnetic result is seen to be a direct extension of the scalar result where intensity fluctuations manifest themselves as the visibility of the interference pattern. This is again an explicit example of the influence of polarization modulation to the electromagnetic coherence.

Furthermore, the HBT effect is examined in special situations where different factorization properties, connected to the cross-spectral purity and the polarization purity phenomena discussed in Chapter 4, are introduced to fields. Additionally, it is shown that the degree of cross-polarization in Eq. (6.4) is a physically problematic measure, as the intensity-fringe visibility $|\eta_0(\mathbf{r}_1, \mathbf{r}_2, \omega)|$ may very well take on zero values even if the investigated fields are correlated. This leads to $\mathcal{P}$ approaching infinity in some situations.

7 Conclusions and outlook

In this thesis, various coherence phenomena related to partially coherent and partially polarized fields have been analyzed by employing the electromagnetic coherence theory. More precisely, the invariance conditions for both the normalized spectrum and the state of polarization in Young's interference experiment have been discussed, the spectral invariance on propagation in free space has been examined, and the electromagnetic Hanbury Brown–Twiss (HBT) effect has been reassessed. The results of these studies have been published in papers **I–V** that are attached at the end of the thesis.

7.1 SUMMARY OF MAIN RESULTS

Paper **I** deals with the electromagnetic extension of Mandel's scalar theory of cross-spectral purity. Cross-spectrally pure fields are characterized by the fact that their normalized intensity spectra remain invariant on superposition in Young's two-pinhole experiment. For electromagnetic fields this condition leads to the factorization of the normalized two-point Stokes parameter $\nu_0(\mathbf{Q}_1, \mathbf{Q}_2, \tau)$ into temporal and spatial parts. Moreover, the absolute value of the normalized spectral two-point Stokes parameter $\eta_0(\mathbf{Q}_1, \mathbf{Q}_2, \omega)$ is independent of frequency and equals $|\nu_0(\mathbf{Q}_1, \mathbf{Q}_2, \tau_0)|$ with some τ_0. These results are analogous to those of the scalar case in the sense that η_0 and ν_0 describe the intensity modulations in Young's interference experiment. However, the class of cross-spectrally pure electromagnetic fields is more complex than its scalar counterpart. For example, the relationship between the cross-spectral purity of the individual field components and the cross-spectral purity of the total vector field is not straightforward.

In paper **II** we examine the cross-spectral purity of the Stokes parameters. This corresponds to a situation where one Stokes parameter S_n remains invariant in Young's interference experiment at every frequency. The invariance of S_n results in conditions similar to those obtained in the analysis of the electromagnetic cross-spectral purity but for the normalized two-point Stokes parameters $\psi_n(\mathbf{Q}_1, \mathbf{Q}_2, \tau)$ and $\mu_n(\mathbf{Q}_1, \mathbf{Q}_2, \omega)$. Moreover, if all the Stokes parameters are cross-spectrally pure regarding the same area in the observation plane, the temporal and spectral degrees of coherence for electromagnetic fields, γ_{em} and μ_{em}, are equal.

Furthermore, the purity of polarization state is examined in paper **III**. We show that the separation of the CSD matrix into parts describing polarization and spatial correlation leads to the invariance of the polarization state on superposition. This is true in both the space–time and space–frequency domains but, due to the differences in the polarization properties of light between these domains, the physical implications are different. For example, a field with pure polarization in the time domain is necessarily pure also in the frequency domain, but the reverse does not hold. Moreover, the separation of polarization and spatial modulation in the case of fields with pure polarization can be seen as a departure from the usual non-quantum entanglement.

In addition to paper **I**, we study the spectral invariance of vector fields also in paper **IV** but this time in the context of propagation in free space. We show that the normalized spectrum of the far field emitted by a statistically homogeneous, planar source equals the normalized spectrum of the source in every direction if the source obeys the electromagnetic scaling law. This scaling law is fulfilled if the trace of source's normalized CSD matrix depends only on the quantity $k\Delta\boldsymbol{\rho}$. As with the concept of cross-spectral purity, the electromagnetic version of the scaling law is also more intricate than its scalar counterpart. This can be seen from the examples illustrated in paper **IV**.

The coherence properties of the vector fields are connected to the intensity correlations in paper **V**. We show that the degree of coherence for electromagnetic fields fully describes the correlations of the intensity fluctuations in the HBT experiment and no other measures are needed. This result is further compared to the earlier analyses where the electromagnetic HBT effect is explained using Wolf's electromagnetic degree of coherence and the degree of cross-polarization. Moreover, we discuss the effects of the polarization state on the HBT experiment, and show that the physical interpretation of the intensity correlations is more complex with partially polarized light than with completely polarized light.

7.2 OUTLOOK

The purity studies described in this thesis provide insight into the special classes of fields whose spectrum or polarization state remains invariant under superposition. In the future it would be intriguing to create cross-spectrally pure electromagnetic fields or fields with pure partial polarization in laboratory conditions. It would also be of interest to investigate the relationship between cross-spectral purity and the scaling law in the case of vector fields. Possible applications for the electromagnetic scaling law could be found from the spectroscopy and characterization of light sources. Additionally, the results obtained in the electromagnetic analysis of the HBT effect can be used, for example, in the studies of coherence vortices, i.e., the singularities of the correlation function, in vector fields [112, 113]. Also an experimental investigation of the effect by measuring the contrast parameters could be considered.

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This thesis contains theoretical studies on different coherence phenomena related to partially coherent and partially polarized fields. The cross-spectral purity of electromagnetic fields is studied, and the concept is extended to cover also the Stokes parameters. Additionally, the polarization purity, or invariance, in Young's experiment is discussed in the time and frequency domains. The spectral invariance of vector fields is investigated also on propagation into the far zone. Moreover, the Hanbury Brown–Twiss experiment is analyzed for vector fields.

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