State-space approach to capital return in nonlinear growth processes

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Structured Abstract

Purpose

We introduce a capital return rate function for growth processes, and apply it to financial sustainability considerations in growing multiannual plants.

Design

A partition function of change rate of capitalization is introduced, as well as that of capitalization itself, and the expected value of capital return rate is produced as the ratio of the two functions.

Findings

Financial sustainability significantly differs from maximum-yield sustainability, and does not depend on any external interest rate.

Research Implications

It is proposed that financial considerations should not be based on any arbitrary external interest. Neither should the shape of any yield function be neglected. Constancy of capital return rate in time is not assumed.

Practical Implications

Two forestry examples show that the capital return rate is sensitive to rotation time, and in particular to the level of initial investment. The proposed procedure can be applied in the absence of periodic boundary conditions in time.

Originality

The methodology has not been applied in this field previously.

Keywords

Multiannual plants; partition function; rotation age; real estate

1 Introduction

Businesses should be sustainable. In businesses involved in growing multiannual plants, sustainability may refer to maintenance of growing stock, maintenance of growth, or maintenance of productive area (Kuusela 1961, Posavec et al. 2012). A variety of decisions contributes to such measures, including rotation times, regeneration practices, eventual thinning schedules, etc. Another view is that maintenance is not necessarily enough, there possibly should be a progression in the amount of growing stock, growth, or possibly productive area (Kuusela 1961).

Financial considerations have been incorporated to sustainability criteria. Most commonly, a discount interest rate is applied in order to compute the present value of future incomes and expenses (Faustmann 1849, Pearse 1967, Samuelson 1976, Yin and Newman 1995, Deegen et al. 2011, Campbell 1999, Nyyssönen 1999, Tahvonen 2016, Gong and Löfgren 2016, Abdallah and Lasserre 2017). The discount interest rate may vary over time (Price 2011, Buongiorno and Zhou 2011, Brazee 2017, Price 2017). Risk of destructive events has been considered as a premium to the discount interest (Loisel 2011, Hyytiäinen and Haight 2010). Evolution of prices, as well as fluctuations in growth and prices may be added (Buongiorno and Zhou 2011, Yin and Newman 1997). Taxation does contribute, as well as personal financies (Koskela and Alvarez 2004, Tahvonen and Salo 1999, Tahvonen et al. 2001). A Hamiltonian formulation is available (Termansen 2007).

Assuming periodic temporal boundary conditions, the computation of the present value of future incomes and expenses can be extended to infinity (Faustmann 1849, Pearse 1967, Samuelson 1976, Yin and Newman 1995, Deegen et al. 2011, Campbell 1999, Nyyssönen 1999, Tahvonen 2016, Gong and Löfgren 2016, Abdallah and Lasserre 2017). However, periodic temporal boundary conditions do not necessarily exist, at least not up to infinity. In such a case, the computation the net present value of future proceeds becomes rather complicated.

The net present value of future proceeds strongly depends on the discounting interest rate. Then, also the financial soundness of any management decision depends on the discounting interest rate (cf. Tait 1987, Posavec et al. 2012, Abdallah and Lasserre 2016). Even if the discounting interest rate may reflect a variety of factors, it generally reflects the cost of capital, either as a borrowing cost or as an opportunity cost of missed alternative investments. Consequently, reference values may be adopted from the financial market. However, at the time of writing, mortgage interest rates within the Eurozone vary 1% to 2%, but an opportunity cost for neglected alternative investments easily becomes 6% to 9%. Such a range of interest rates dramatically changes net present values, as well as the management decisions considered financially sustainable. The reference range is far too wide.

A third possible problem with the computation of the net present value of future proceeds is that it does not consider capitalization in any way. We suspect there are circumstances where this does not induce problems, but we also suspect there are cases where problems do arise. This issue will be discussed in more detail below.

We are aware of one process for the determination of rotation age without a predetermined discount rate (Boulding 1955, Newman 1988). A rotation time is found that maximizes internal discount interest, harvesting income being discounted in order to cover initial investments (Boulding 1955, Newman 1988).

Instead of discounting final income, we discuss capital return rate in multiannual nonlinear growth processes. In order to compose an expected value of the capital return rate, we introduce a state-space approach to the capital return. We define a partition function of momentary capitalization, as well as that of a momentary return rate of capital. An expected rate of capital return is produced as the ratio of the two functions.

In the special case of constant capital return, the outcome naturally shall correspond to exponential prolongation of initial investment. However, in the case of real-life growth processes, the momentary capital return rate typically varies according to some growth function, and also depends on capitalization.

As a practical example of the different capital return approaches, we discuss the optimal rotation time in growing multiannual plants. We compare the state-space results with some alternative financial sustainability criteria. In particular, the outcome is compared with the criterion of maximum internal rate of return (Boulding 1955, Newman 1988). The latter criterion is special because it does neither contain any arbitrarily chosen external discount rate. The comparison is implemented in terms of two different yield functions (Gong and Löfgren

2016), along with a few parametrizations. Finally, prospective further applications are discussed.

2 State-space capital return model

Let us first introduce a three-dimensional continuous state space, the three dimensions consisting of capitalization per unit area, change rate of the capitalization, and time. We first write a partition function of the capitalization K within a particular time range

$$Z_K = \int_0^{\tau} K(t)dt \qquad (1).$$

Then we write the partition function of the time change rate of capitalization

$$Z_{\frac{d\kappa}{dt}} = \int_0^{\tau} \frac{d\kappa}{dt} dt \qquad (2).$$

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The expected value of capitalization is

$$E[K] = \int Kp(K)dK = \int_0^{\tau} Kp(K) {}^{dK}/_{dt} dt = \int_0^{\tau} K(t)p(t)dt = {}^{Z_K}/_{\tau} \qquad (3),$$

where p refers to a probability density function.

Similarly. the expected value of the change rate of capitalization is

$$E\left[{}^{d\kappa}/_{dt}\right] = \int {}^{d\kappa}/_{dt}\, p({}^{d\kappa}/_{dt}) d({}^{d\kappa}/_{dt}) = \int_0^{\tau} {}^{d\kappa}/_{dt}\, p(t)dt = {}^{Z_{\frac{d\kappa}{dt}}}/_{\tau} \qquad (4).$$

The expected value of the change rate of capitalization, in relation to the expected value of capitalization, corresponds to the expected value of the capital return rate

$$E[r] = {}^{Z_{\frac{d\kappa}{dt}}}/_{Z_K} \qquad (5).$$

Considering that the momentary capital return rate is

$$r = \frac{d\kappa}{Kdt} \qquad (6),$$

the partition function of the change rate of capitalization can be rewritten

$$Z_{\frac{d\kappa}{dt}} = \int_0^{\tau} K(t) r(t) dt \qquad (7),$$

and substituted into Eq. (5) at will.

In order to reflect capital return rate within the system, the change rate of capitalization $\frac{d\kappa}{dt}$ in the above equations must correspond to changes occurring internally, in terms of growth, eventual amortizations, etc. In an open system, however, capital flows may appear from (or to) the environment through investments or withdravals. The capitalization K must thus consider any eventual exchange of capital with the environment.

The simplest possible application of the above Equations would correspond to a case with constant capital return rate and non-amortizable initial investment. In that case the capitalization would simply correspond to exponential prolongation of the initial capitalization, and the expected value of the capital return rate would become

$$E[r] = \frac{\left[e^{r\tau} - 1\right]}{\int_0^{\tau} e^{rt} dt} = r \qquad (8).$$

Real-life yield curves however often significantly differ from simple exponential prolongation. We will below discuss some yield curves in detail. However, let us first briefly compare Eq. (5) with techniques traditionally used for the evaluation of financial sustainability of growing multiannual plants like forest trees.

Firstly, the seminal approach introduced by Faustmann 170 years ago, simply computes the present value of all expenses and revenues as (Faustmann 1849, Pearse 1967, Samuelson 1976, Yin and Newman 1995, Deegen et al. 2011, Campbell 1999, Nyyssönen 1999, Tahvonen 2016, Gong and Löfgren 2016, Abdallah and Lasserre 2017)

$$NPV_{t=0} = \int_0^{\tau} R(t) e^{-qt} dt \frac{1}{1 - e^{-q\tau}} \qquad (9),$$

where where $R(t)$ corresponds to net proceeds at time t, q is discounting interest rate, and τ again is rotation age. The latter factor in Eq. (9) arises from discounting of further growth cycles, each of duration τ. In other words, Eq. (9) assumes a periodic boundary condition in time. A significant problem in Eq. (9) is that it contains an external discount rate q. The discount rate is external in the sense that it is unrelated to the capital return within the growth process, and consequently subject to arbitrary changes. Another issue in Eq. (9) is that it does not consider the shape of the yield curve in any way. In other words, Eq. (9) discusses revenue in cash basis, instead of financial grounds. In the absence of investments or withdravals within any growth period, Eq. (9) can be rewritten

$$NPV_{t=0} = \left[K(\tau)e^{-q\tau} - K(0)\right]\frac{1}{1-e^{-q\tau}} \qquad (10).$$

The problem of arbitrary external interest has been resolved by introducing an internal rate of return. As introduced by Newman (Newman 1988, Boulding 1955), it is determined for a period of duration τ according to the criterion

$$\int_0^\tau R(t)e^{-st}dt = 0 \qquad (11),$$

where s is internal rate of return. In the absence of investments or withdravals within any growth period, Eq. (11) can be rewritten

$$K(\tau)e^{-s\tau} - K(0) = 0 \qquad (12).$$

Ihere is no explicit indication of further growth cycles in Eqs. (11) or (12). These Equations however do apply to cyclical boundary conditions in time, but unlike Eqs. (9) and (10), such boundary conditions are not required. From Eq. (12), the internal rate of return can be readily resolved as

$$s(\tau) = \frac{1}{\tau}\ln\left(\frac{K(\tau)}{K(0)}\right) \qquad (13).$$

A substantial benefit of Eq. (11), in comparison to Eq. (9) is that it does not contain any arbitrary external interest. Eq. (11) is not solvable in the absence of any investment. This, however, is not too detrimental since there hardly is any production that would not require investments, at least in terms of the opportunity cost of not selling bare production land. Eq. (12) neither considers the shape of the yield curve.

3 A practical forestry example with volumetric growth

As a practical example, we consider a recently introduced (Gong and Löfgren 2016) yield function, applicable to average pine stands in Northern Sweden. A volumetric growth function is

$$V(t) = 580.14 * \left(1 - 6.3582^{-t/95}\right)^{2.8967} \qquad (14).$$

The application introduced by Gong and Löfgren (2016) assumes a volumetric stumpage price of 250 SEK/m^3, and an initial investment of 6000 SEK/ha. The maximum sustained yield rotation being 95 years, corresponding to that duration of time that gives the greatest average annual growth (Gong and Löfgren 2016). A 3% discount interest applied in Eq. (10), without considering any bare land value, yields an optimal rotation age of 52 years (Gong and Löfgren 2016). We now apply Eq. (5) for this case.

Fig. 1 shows the value growth function, in terms of momentary growth, average growth and accumulated growth. The Figure confirms that the maximum sustained yield is reached at rotation time 94 years: this is where momentary and average growth curves coincide.

Fig. 1 does not contain any amortization of investments, nor any bare land value. These factors are introduced in Fig. 2. The base case is that the initial regeneration investment of 6000 SEK/ha, amortized at the end of any rotation. A non-amortizable bare land value of 2500 SEK/ha is included in the capitalization. Fig. 2 shows the partition function of capitalization (Eq. (1)), as well as the partition function of the time change rate of capitalization (Eq. (2)). While the capitalization per hectare is always positive, the state sum of the time change rate is negative until rotation age 21. This is due to the amortization of the initial investment of SEK 6000/ha. Correspondingly, the expected value of capital return rate, according to Eq. (5), is negative up to rotation age 21, and reaches its maximum at rotation age 48.

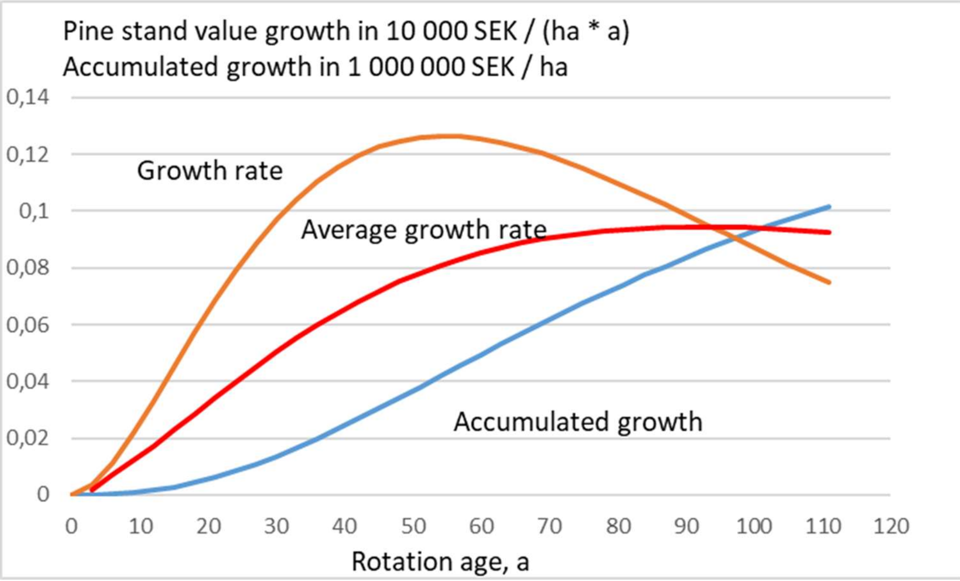

Fig. 1. Pine stand value growth according to a North-Swedish growth function (14), (Gong and Löfgren 2016).

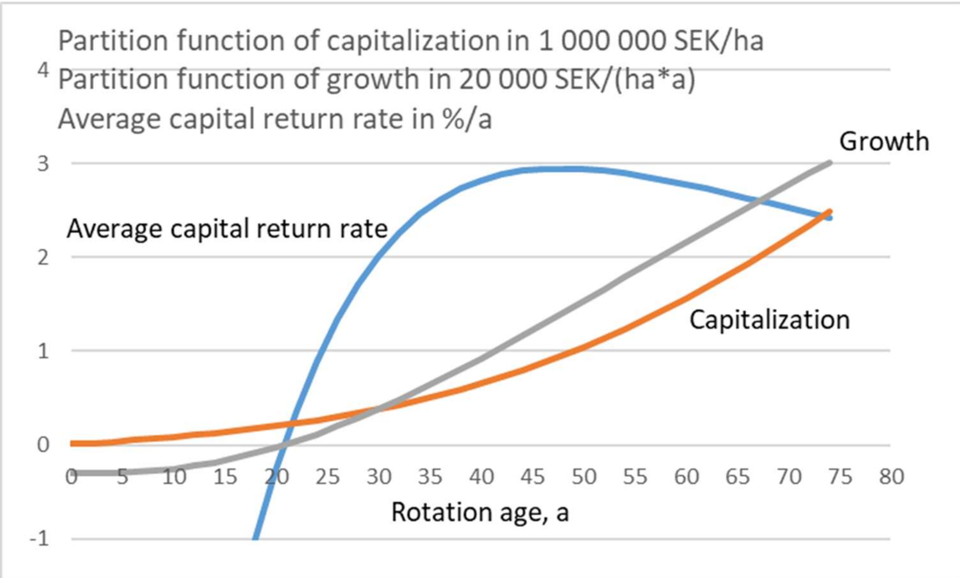

Fig. 2. Partition function of capitalization according to Eq. (1), and growth according to Eq. (2), as well as expected value of capital return rate according to Eq. (5), as a function of rotation age.

Figure 3 shows the expected value of the capital return rate according to Eq. (5) and the internal rate of return according to Eq. (13) as functions of rotation age. In addition, Figure 3 shows the net present value of further growth according to Eq. (10), using 2% and 3% discounting interest rates. Again, a non-amortizable bare land value of 2500 SEK/ha is included in the capitalization. The average capital return rate reaches its maximum at rotation age of 48 years. The internal rate of return reaches maximum at 52 years. 2% discounting interest results as maximum net present value at 60 years. 3% discounting interest results as maximum net present value at 52 years.

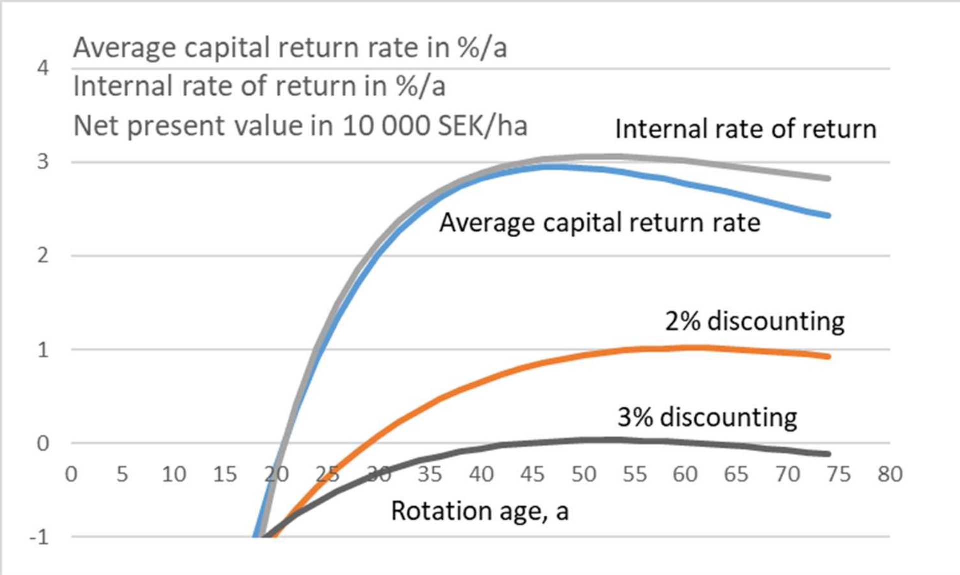

Fig. 3. Capital return rate (Eq. (5)), internal rate of return (Eq. (13)) and net present value of proceeds (Eq. (10)) as functions of rotation age. The NPV is computed with 2% and 3% discounting interests.

4 A practical forestry example with value growth

Eq. (10), as well as Fig. 1, assumes the volumetric stumpage price to be constant. In other words, the value growth corresponds to volumetric growth, multiplied by a constant. That may be an unrealistic assumption, for a variety of reasons, including harvesting expenses as well as industrial use of the crop. In order to release this assumption, Gong and Löfgren (2016) established an age-dependent price function

$$h(t) = 104.63 * (t - 29)^{0.2602} \qquad (15).$$

We will now apply Eq. (15), for t>29, in addition to Eq. (14), in order to establish another version of the practical forestry example.

Fig. 4 shows the outcome. Maximum sustainable value yield is gained at 130 years of rotation. There is a break-even point at 31 years, where internal interest becomes nonnegative, according to Eq. (9).

Fig. 4 does not contain any amortization of investments, neither any bare land value. These factors are introduced in Fig. 5. Again, the base case is that the initial regeneration investment of 6000 SEK/ha amortized at the end of any rotation. A non-amortizable bare land value of 2500 SEK/ha is included in the capitalization. Fig. 5 shows the partition function of capitalization (Eq. (1)), as well as the partition function of the time change rate of capitalization (Eq. (2)). While the capitalization per hectare always is positive, the state sum of the time change rate is negative until rotation age 31. This is due to the amortization of the initial investment of SEK 6000/ha. Correspondingly, the expected value of capital return rate, according to Eq. (5), is negative up to rotation age 31, and reaches its maximum at rotation age 53.

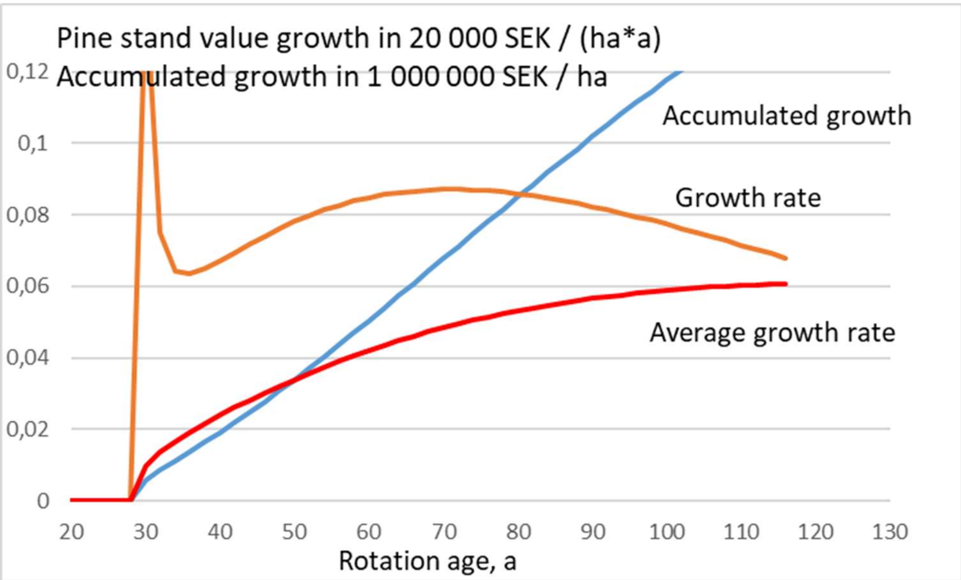

Fig. 4. Pine stand value growth according to a North-Swedish value growth function (14), (Gong and Löfgren 2016).

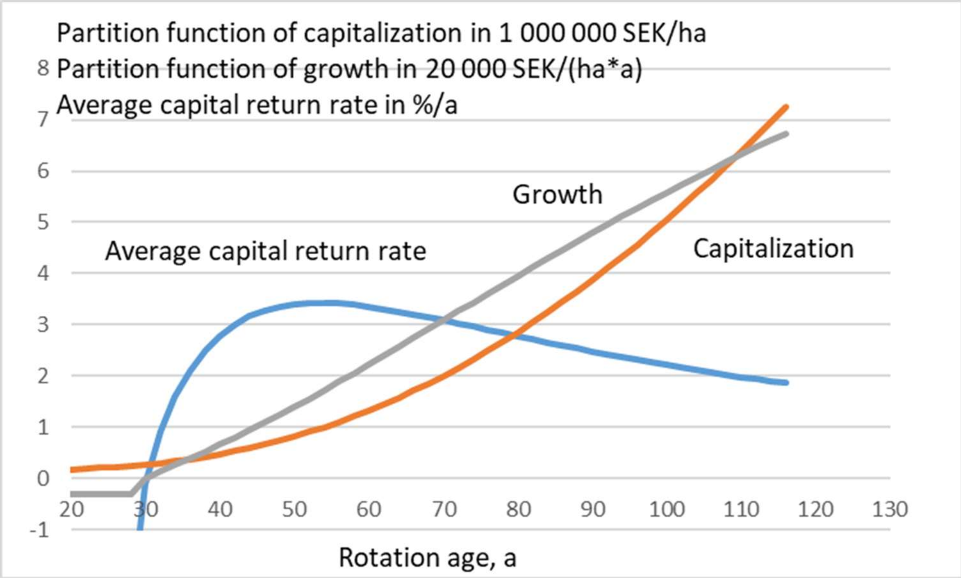

Fig. 5. Partition function of capitalization according to Eq. (1), and growth according to Eq. (2), as well as expected value of capital return rate according to Eq. (5), as a function of rotation age.

Figure 6 shows the expected value of the capital return rate according to Eq. (5) and the internal rate of return according to Eq. (13) as functions of rotation age. In addition, Figure 6 shows the net present value of further growth according to Eq. (10), using 2% and 3% discounting interest rates. Again, a non-amortizable bare land value of 2500 SEK/ha is included in the capitalization. The average capital return rate reaches its maximum at rotation age of 53 years. The internal rate of return reaches maximum at 62 years. 2% discounting interest results as maximum net present value at 72 years. 3% discounting interest results as maximum net present value at 62 years.

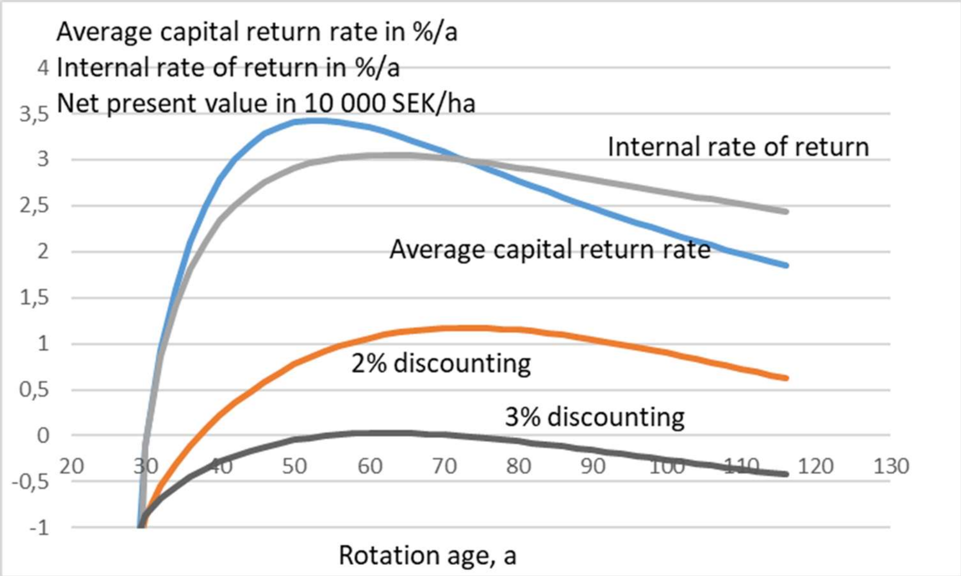

Fig. 6. Capital return rate (Eq. (5)), internal rate of return (Eq. (13)) and net present value of proceeds (Eq. (10)) as functions of rotation age. The NPV is computed with 2% and 3% discounting interests.

5 Further applications

Forest estates are sold and bought at the real estate market. It would be interesting to consider what kind of an investment a young stand might be. The result naturally depends on the purchase price. In order to create an example, let us discuss the North-Swedish case defined by Eqs. (10) and (11) to be purchased at the age of 20 years. Now, there are no periodic boundary conditions in time. This makes Eqs. (9) and (10) inapplicable, whereas Eqs. (5), (10) and (11) can be applied.

We find from Fig. 7 that if the stand is acquired at the price corresponding to the sum of the initial investment of 6 000 SEK/ha and the bare land value of 2 500 SEK/ha, a capital return rate of 4.4% is gained if harvested at age 44, according to Eq. (5). The maximum internal rate of return according to Eq. (11) is 4.9%, gained at age 46. An acquiry price of 12 000 SEK/ha would correspond to capital return 3.2% at age 52, and internal rate of return of 3.7% at age 54. An acquiry price of 18 000 SEK/ha would correspond to capital return 2.3% at age 64, and internal rate of return of 2.7% at age 66.

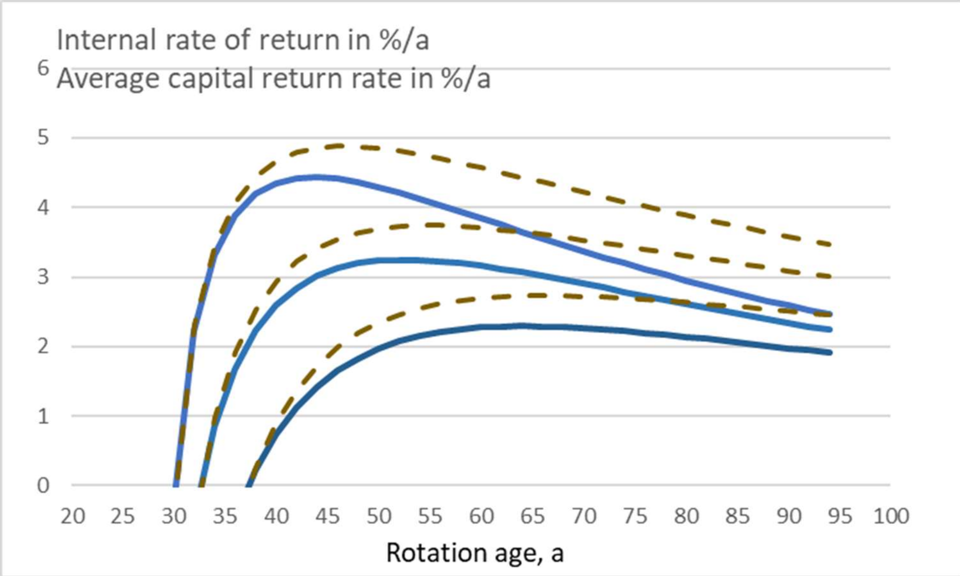

Fig. 7. Pine stand value growth according to a North-Swedish value growth function (11), (Gong and Löfgren 2016). Solid lines correspond to average financial return rate, as a function of rotation time τ, according to Eq. (5), for three different levels of acquiry price at the age of 20. Dotted brown lines correspond to internal rate of return according to Eq. (12).

6 Discussion

A state-space approach for capital return in growing multiannual plants like forest trees was introduced. A partition function of growth rate, as well as capitalization, was introduced. It was shown that the expected value of capital return rate corresponds to the ratio of the partition functions. The outcome does not depend on any external interest rate, which reduces arbitrary factors in any analysis. Provided an applicable yield function can be established for local circumstances, the introduced return rate function can be used in the design of technical operations and commercial transactions.

The introduced state-space approach is not the only procedure not relying on arbitrary external interest. Terminal harvesting income may be discounted to the time of initial investment according to Eq. (11), in order to yield an internal rate of return (Newman 1988, Boulding 1955). However, Eq. (11), as well as Eq. (12), contain an inherent assumption that the rate of return is constant in time. There actually is an equivalency between Eqs. (12) and (5)

$$K(0)\left[e^{s\tau}-1\right]=Z_{\frac{d\kappa}{dt}}=\int_0^\tau K(t)r(t)dt=K(0)\left[\exp\left(\int_0^\tau r(t)dt\right)-1\right] \qquad (16).$$

Clearly, s in Eq. (12) and r in Eq. (7) are two different quantities, as also is visible in Figs. 3, 6 and 7. They coincide only if r becomes constant in time, which is unlikely in the case of the production of multiannual plants. The relation of the two quantities certainly depends whether there is positive or negative covariance between the capitalization K and the momentary capital return rate r.

In order to further clarify the practical differences between the two quantities s and r, let us discuss a distribution of growing stands of different ages. Then, the momentarily expected value of capital return rate is

$$E\left[r(t)\right]=\frac{E\left[\frac{d\kappa}{dt}\right]}{E\left[K\right]}=\frac{\int_0^\tau p(a)\frac{d\kappa(a,t)}{dt}da}{\int_0^\tau p(a)K(a,t)da}=\frac{\int_0^\tau p(a)\kappa(a,t)r(a,t)da}{\int_0^\tau p(a)K(a,t)da} \qquad (17),$$

where $p(a)$ corresponds to probability density function of stand age a. Clearly, one can expect that the momentarily expected return rate of capital depends on the distribution of stand ages. A special case would be constant stand age distribution, which however would recover Eq. (5):

$$E\left[r\right]=\frac{\int_0^\tau \frac{d\kappa(a)}{dt}da}{\int_0^\tau K(a)da}=Z_{\frac{d\kappa}{dt}}\Big/Z_K \qquad (18).$$

Reliance on arbitrary external interest is not the only problem related to the net present value computation given in Eqs. (9) and (10). Apparently, these Equations only apply to periodic boundary conditions in time. Such boundary conditions may at first appear as a convenient idealization. However, there are many situations, including that discussed in Fig. 7, where no such condition exists. Common sense might suggest that in a rapidly changing world, periodic boundary conditions probably do not often exist for systems growing several decades.

It is often stated that an external discounting interest rate reflexes the cost of capital, which in turn depends on the risks involved (Pearse 1967, Samuelson 1976). A question arises, how the risks of destructive events are considered in the state-space approach to capital return.

Firstly, let us discuss risks internal to the growth process: risks depending on selection of plant species and regeneration practices, application of pre-commercial and commercial thinnings, drainage, fertilization, etc. Such risks possibly should be included in the yield function as stochastic elements. This is far from trivial, and obviously should be applied to any yield function regardless of the economic objective function used.

Risks of externally introduced destructive events may or may not be embedded in the discounting interest, while computing a net present value (Loisel 2011, Hyytiäinen and Haight 2010). A stochastic risk formulation can be included in the objective function, regardless if it is based on net present value, internal rate of return, or an expected value of capital return rate.

In financial theory, high-risk investments or investment environments require a higher expected return rate, or a greater discounting interest rate. The internal rate of return in Eqs. (11) and (12) maximizes the discounting interest. The state-space approach (Eq. (5)) maximizes the expected rate of capital return. There is no artificial interest rate that could be manipulated on the basis risk analysis. On the other hand, in computation of net present value according to Eqs. (9) or (10), there is an external discounting interest that can be manipulated on the basis of risk analysis. The result of the manipulation in the occurrence of strongly increasing risk factors would be that the discounting interest would approach the rate of internal return, in which case equations (9) and (11) would coincide.

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