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Ultrasound field characterization using synthetic schlieren tomography

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Synthetic schlieren imaging, also known as background oriented schlieren imaging, is used to determine the acoustical field of a focused ultrasound transducer operating at 1.01 MHz frequency with peak pressure amplitude of 0.97 MPa. The measurement setup is composed of a commercial off-the-shelf digital single-lens reflex (DSLR) camera with an ordinary objective, a high power light-emitting diode driven in pulsating mode, water tank, ultrasound transducer, rotation stage, and driving electronics. Measurements are performed in tomographic fashion by rotating the ultrasound transducer within the water tank and photographing an imaged target behind the ultrasound field. The photographs are processed with a Horn-Schunck-type algorithm, commonly used in optical flow analysis, in order to determine the deflection of light rays as caused by ultrasound field induced acousto-optic effect. Inverse Radon transform is then used, with the deflection data, to obtain three-dimensional spatial distribution of the pressure field gradient, from which an approximation of the ultrasonic pressure field is computed. The pressure field obtained with synthetic schlieren tomography is then compared to hydrophone measurements mainly qualitatively.

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I. INTRODUCTION

Ultrasound field characterization and calibration is becoming increasingly important. Sophisticated medical imaging techniques, such as those reliant on computational full-wave inversion or those that seek to map measured signals into quantitative parameters of interest, require calibration of ultrasound transducers in order to accurately map measured signals into physical quantities. In ultrasound therapy, knowledge of the shape and intensity of the ultrasound field, as produced by a therapy device, is crucial for efficient and safe treatment planning and delivery. When these devices are composed of hundreds or thousands of individual transducer elements, the characterization of the devices can become cumbersome. In this work, the feasibility of an optical technique is investigated for three-dimensional (3D) characterization of the ultrasound field.

Several methods to characterize, visualize, and calibrate acoustic fields of an ultrasound transducer exist. Measurements using a needle, or a membrane, hydrophone are regarded as the golden standard method. Present hydrophones, utilizing piezoelectric polyvinylidene fluoride (PVDF) membranes, enable good sensitivity at a wide frequency bandwidth. However, hydrophones are expensive and delicate instruments, and can require calibration at regular intervals. They are sensitive to mechanical shocks, acoustic cavitation-induced erosion, as well as electromagnetic interference. In addition, operating a hydrophone can require great accuracy, patience, and can be time consuming. In a typical hydrophone measurement setup, the ultrasound transducer and the hydrophone are first carefully aligned with respect to each other. Then, the hydrophone is spatially moved from point to point in the acoustic field, and a waveform resembling the acoustic pressure is recorded at each point with an oscilloscope. In the case of complex acoustic sources, or repeated measurements, hydrophone measurements can be laborious.

Several optical methods have been presented as an alternative, or as a supplement, to hydrophones. Fiber-optic hydrophones, utilizing either Fresnel refraction or Fabry-Pérot interferometry, can be used in harsh environment, and offer a reduced directional sensitivity. The measurement protocol is similar to hydrophone, however. Optical interferometry and laser Doppler vibrometry used in hydrophone calibrations can be used as an alternative for hydrophones. In a scan, a laser beam is focused on a light reflecting and acoustically transparent membrane and the displacement or velocity amplitude of the membrane due to the acoustic wave is measured point by point. Vibrometer equipped with a scanning laser head can be used for more rapid measurements over two-dimensional (2D) planes. In non-perturbing vibrometer method, the laser signal propagates through the acoustic field and reflects from the opposite mirror back to the vibrometer. The measured rate of change in the optical path length in the acoustic field can be used to identify the acoustic beam pattern. Thermometry imaging, using, e.g., an infrared camera, can be used to measure ultrasound-induced temperature rise in an ultrasound absorber, with the temperature rise corresponding to spatial shape and amplitude of ultrasonic intensity distribution.

Schlieren methods can offer higher measurement speeds in comparison to methods where individual points or planes are measured. It is possible to visualize whole acoustic fields in real time, without disturbing the ultrasound fields mechanically, using the methods. Schlieren methods are also immune to electromagnetic interferences.

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Background-oriented schlieren (BOS) imaging is a technique based on diffraction of light as it propagates through a heterogeneous distribution of refractive index.\textsuperscript{15} In the technique, photographs of a background image are taken under homogeneous and heterogeneous conditions of refractive index field, and the photographs are then used as data in optical flow algorithms.\textsuperscript{16} When the light recorded by the camera originates from the imaged target, these algorithms result in 2D displacement images reflecting projections of the gradient of index of refraction in the plane perpendicular to the direction of light propagation. Since ultrasound can cause a spatial perturbation of index of refraction due to acousto-optic effect, the method can be utilized in ultrasonics. Previously, a similar approach based on imaging a background target has been used in the ultrasound community and has been referred to as synthetic schlieren imaging,\textsuperscript{17} visualizing ultrasound field with focused shadowgraphy,\textsuperscript{18} as well as BOS imaging and tomography.\textsuperscript{19} Synthetic schlieren terminology is adopted for this study.\textsuperscript{17,20,21}

In this work, synthetic schlieren imaging of ultrasound fields is extended to synthetic schlieren tomography (SST) for 3D characterization of ultrasound fields. This is achieved by performing synthetic schlieren imaging at various rotation angles of an ultrasound transducer, and combining the data at multiple angles to obtain the 3D ultrasound field. The SST measurements are obtained by using relatively simple hardware composed of a pulsed light-emitting diode (LED) that is temporally synchronized with a focused ultrasound transducer, a commercial off-the-shelf digital camera, measurement tank, and the related LED and ultrasound driving electronics. For the analysis of the data, a Horn-Schunck-type optical flow algorithm\textsuperscript{22} is used to obtain displacement images of the refracted light as caused by the ultrasound. Following this, inverse Radon transform\textsuperscript{23,24} is used to compute a 3D image of the derivative of the ultrasound field, and an approximate method is then used to form the 3D pressure field. The presented approach is tested by performing measurements of a focused ultrasound transducer. The resulting pressure field is then compared to measurements done using a needle hydrophone.

II. MATERIALS AND METHODS

The principle of SST is depicted in Fig. 1. Figure 1 depicts a camera, a light source placed behind an imaged target, medium undergoing perturbation of refractive index due to a ultrasound field, refracted rays of light, and observed unperturbed and perturbed photographs. Figure 1 also depicts the coordinate system used throughout the paper. In Sec. II A, the theory behind SST is described, followed by Secs. II B and II C describing the measurement setup used in this work, as well as the measurements and data processing performed.

A. Theory

When small amplitude ultrasound beam propagates through an isotropic dielectric medium, such as water, the change of refractive index, \( n \), with respect to the acoustical pressure can be described with linear approximation\textsuperscript{25,26}

\[
n(x, y, z, t) = n_0 + \left( \frac{\partial n}{\partial p} \right) p(x, y, z, t),
\]

where \( n_0 \) is the refractive index of the ambient medium, \( p(x, y, z, t) \) is the acoustic pressure of the ultrasound as a function of space \( (x,y,z) \) and time \( t \), and \( (\partial n/\partial p) \) is the adiabatic piezo-optic coefficient.\textsuperscript{27,28} The change of refractive index due to sound is called the acousto-optic effect.

Propagation of rays of light, originating from \( r = (x,y) \) behind a small perturbation and initially propagating toward positive \( z \) axis through a field of varying refractive index, see Fig. 1, can be modeled using\textsuperscript{15,21,29–31}

\[
\begin{align*}
\phi_x(r) &= \frac{1}{n_0} \int \frac{\partial n}{\partial z} (x, y, z) \, dz, \\
\phi_y(r) &= \frac{1}{n_0} \int \frac{\partial n}{\partial y} (x, y, z) \, dz,
\end{align*}
\]

where \( \phi_x(r) \) and \( \phi_y(r) \) are the horizontal and vertical deflection (in radians) of the light ray from \( z \) axis toward \( x \) and \( y \) axes, respectively, the integration is carried over a line through the perturbed volume, and it has been assumed that the variations of the refractive index are located in a small volume. As the deflected light ray continues propagation past the perturbation for a distance \( D \) along the \( z \) axis, the displacement of the light ray originating from \( r \) can be expressed for small deflections as

\[
\begin{align*}
u(r) &= D \phi_y(r), \\
v(r) &= D \phi_x(r),
\end{align*}
\]

where \( u(r) \) and \( v(r) \) are the horizontal and vertical displacement of the ray of light that is observed at position \( r + \delta(r) \), respectively, with \( \delta(r) = (u(r), v(r)) \). Assuming that the light ray propagates through the ultrasound field instantaneously at time \( t \), the displacement of light rays due to ultrasound can be expressed using Eqs. (1)–(3) as

\[
\begin{align*}
u(r, t) &= \kappa \int \frac{\partial p}{\partial x} (x, y, z, t) \, dz, \\
u(r, t) &= \kappa \int \frac{\partial p}{\partial y} (x, y, z, t) \, dz,
\end{align*}
\]

where \( \kappa = (D/n_0)(\partial n/\partial p) \) is the proportionality term relating the projections of pressure gradient to absolute displacement.
Since the speed of light in, e.g., water is about 151 000 faster than the speed of ultrasound, phase of the ultrasound field changes negligibly during the time it takes for light to propagate for a medically relevant distance. In order to avoid temporal averaging of the pressure gradient projections [Eq. (4)], pulsed light sources, with the pulse duration shorter than the cycle period corresponding to peak ultrasonic frequency, has to be used.

1. Estimating pressure gradient projections

The relation between an image obtained without ultrasound sonication, denoted by \( I(0) \), and a displaced image obtained during sonication, denoted by \( I^d(0) \), can be modeled as

\[
I^d(r, t) = I(r + \delta(r, t)),
\]

where \( \delta(r, t) = (u(r, t), v(r, t)) \), and it has been assumed that the images are obtained under similar illumination and exposure conditions. Optical flow algorithms, such as Lucas-Kanade, Horn-Schunck, and cross-correlation techniques, can be used to determine \( \delta(r, t) \) given \( I(0) \) and \( I^d(0) \). For comparison of various optical flow algorithms, see, e.g., Refs. 16, 34, and 35. In this work, a Horn-Schunck-type algorithm is used to form estimates for \( \delta(r, t) \) due to its performance in estimating continuous displacements based on noisy data.

The displaced image \( I(r + \delta(r, t)) \) can be approximated by a truncated Taylor series as

\[
I(r + \delta(r, t)) \approx I(r) + u(r, t) \frac{\partial I}{\partial x}(r) + v(r, t) \frac{\partial I}{\partial y}(r),
\]

where \( I(r) \) corresponds to photograph without ultrasound sonication. Omitting the explicit dependence on \( r \) and \( t \), the displacement field is estimated with Horn-Schunck-type regularized least squares

\[
(\hat{u}, \hat{v}) = \arg \min \left\{ \left| I^d - I - u \frac{\partial I}{\partial x} - v \frac{\partial I}{\partial y} \right|^2 + \sigma^2(\nabla^2 u)^2 + (\nabla^2 v)^2 \right\} dr,
\]

where \( \hat{\delta} = (\hat{u}, \hat{v}) \) is an estimate of \( \delta \), the 2D integration is carried over the imaged area, and \( \sigma \) is a regularization parameter governing how much weight should be put on the Laplacian regularization term in comparison to data misfit term.

In practice, \( I \) and \( I^d \) are obtained using a camera and are discrete digital images composed of \( Q \) pixels. Expressing these images as vectors \( \mathbf{I} = (I_1, ..., I_Q)^T \) and \( \mathbf{I}^d = (I_{q1}, ..., I_{qQ})^T \), where \( I_q \) and \( I_{qj} \) for \( q = 1, ..., Q \), are the pixel intensities of the discrete photographs, the regularized least squares [Eq. (7)] can be written in discrete form as

\[
(\hat{u}, \hat{v}) = \arg \min \left\{ \left| \mathbf{I}^d - I - D_u \mathbf{u} - D_v \mathbf{v} \right|^2 + \sigma^2(\left| \mathbf{L_u} \right|^2 + \left| \mathbf{L_v} \right|^2) \right\},
\]

where \( \mathbf{u} = (u_1, ..., u_Q)^T, \mathbf{v} = (v_1, ..., v_Q)^T \), \( \| \cdot \| \) is the Euclidean 2-norm, \( D_u = \text{diag}\{I_{x1}, ..., I_{Qx}\} \) and \( D_v = \text{diag}\{I_{y1}, ..., I_{Qy}\} \) are diagonal matrices with \( I_{xq} \) and \( I_{yq} \) being first-order centered finite difference approximations of \( x \) and \( y \) derivatives of \( I \) at pixel \( q \), and \( L \) is a matrix corresponding to discrete Laplacian operator based on second-order centered finite differences. For details on computation of finite differences, see Ref. 36. Solution of the discrete regularized least squares of form Eq. (8) can be obtained using

\[
\begin{align*}
\begin{pmatrix}
\hat{u} \\
\hat{v}
\end{pmatrix} &= A^{-1}b,
\end{align*}
\]

where

\[
A = \begin{pmatrix}
D_x^T D_x + x^2 L_x^T L_x & D_x^T D_y + x^2 L_y^T L_x \\
D_y^T D_x + x^2 L_x^T L_y & D_y^T D_y + x^2 L_y^T L_y
\end{pmatrix},
\]

and

\[
b = \begin{pmatrix}
D_x^T (I^d - I) \\
D_y^T (I^d - I)
\end{pmatrix}.
\]

2. Approximating pressure field from tomographic data

Given that the ultrasound transducer is rotated in \( xz \) plane around \( y \) axis by angle \( \theta \), the pressure field \( p \) in laboratory coordinates \( (x, y, z, t) \) can be mapped from ultrasound transducers’ local coordinate system \( (x', y', z', t') \) using

\[
\begin{align*}
x &= x' \cos(\theta) - z' \sin(\theta), \\
y &= y', \\
z &= x' \sin(\theta) + z' \cos(\theta), \\
t &= t'.
\end{align*}
\]

This implies that the displacement of light rays [Eq. (4)] in laboratory coordinates become

\[
u(r, \theta, t) = \kappa \int \frac{\partial p}{\partial x'} \frac{\partial x'}{\partial x} + \frac{\partial p}{\partial z'} \frac{\partial z'}{\partial z} dz
\]

and

\[
v(r, \theta, t) = \kappa \int \frac{\partial p}{\partial y'} \frac{\partial y'}{\partial y} dz = \kappa \int \frac{\partial p}{\partial y} dz,
\]

where \( p = p(x', y', z', t') \) is the pressure field of the ultrasound transducer in its local coordinate system.

While estimation of \( p \) (or its derivatives in \( xz \) plane) from Eq. (13) can be difficult, in this work the ultrasound pressure is approximated based on \( \partial p/\partial y \) that can readily be estimated from Eq. (14). Since \( v(r, \theta, t) \) directly reflects projection (line integral) of the \( y \)-derivative of pressure, inverse Radon transform (using filtered back-projection algorithm) is used in this work to estimate the derivative of the pressure field.

In order to obtain \( p \) from \( \partial p/\partial y \), some approximations have to be made. The linear wave equation for propagation
of sound in lossless medium for continuous wave, such as a long ultrasound pulse in water, is
\[ \nabla^2 p + k^2 p = 0, \tag{15} \]
where \( k = \omega c \) is the wavenumber, with \( \omega \) being the angular frequency of ultrasound, and \( c \) is the speed of sound. A plane wave approximation, in direction of \( y \) axis, results in the one-dimensional (1D) wave equation
\[ \frac{\partial^2 p}{\partial y^2} + k^2 p = 0. \tag{16} \]
From Eq. (16) an approximation for pressure is obtained as
\[ p \approx -\frac{\omega^2}{c^2} \frac{\partial}{\partial y} \left( \frac{\partial p}{\partial y} \right). \tag{17} \]
Thus, the pressure \( p \) can be approximated by computing the \( y \)-derivative of \( \frac{\partial p}{\partial y} \), which is obtained using inverse Radon transform.

**B. Measurement setup**

Figure 2 shows the sketch of the measurement setup used in this work. The setup was constructed around a glass tank (Lasilinkki Oy, Kuopio, Finland) made of 5 mm thick aquarium-grade glass and placed on a rigid stone table. The tank had external dimensions \( 29.6 \times 29.6 \times 44 \text{ cm} \) (width \( \times \) depth \( \times \) height) and was filled with degassed deionized water (temperature \( T = 21.6 \degree C \), corresponding speed of sound \( c = 1487 \text{ m/s} \); Ref. 40) during the experiments. The bottom of the tank was covered with a Aptflex F28 acoustic absorber (Precision Acoustics, Dorchester, UK) and pimpled rubber mats to eliminate acoustic reverberations.

On one side of the tank, the imaged target was held in place on the outer side of the tank wall by a 1.7 mm thick glass plate. Pulsed LED illuminating the imaged target from outside the tank, and its driving circuitry, were attached to a laboratory stand support and positioned 41 mm away from the glass plate at the rough center of the imaged target.

On the opposite side of the tank, an entry-level DSLR camera (model EOS 100D camera and model EF-S18–55 mm f/3.5–5.6 IS STM objective, Canon, Japan) was placed 412 mm from the outer wall of the tank opposite of the LED and connected to a computer with universal serial bus (USB). Either an ordinary workstation running Microsoft (Redmond, CA) Windows 7, or a laptop running Canonical (London, UK) Ubuntu 14.04 Linux was used during the measurements. The camera was remotely operated using freely available gphoto2-program (Windows version 2.4.14, Linux version 2.5.3; Ref. 45). The main camera settings were as follows: image size \( 5184 \times 3456 \text{ pixels} \), ISO 200, shutter speed 1 s, focal length 55 mm, \( f \)-number 5.6, and the photographs were saved into minimally processed raw image format. During the measurements, the laboratory was dimmed, so that the camera only captured the imaged target by the illumination of the pulsed LED. Due to the long shutter speed of the camera, the photograph is exposed to multiple light pulses.

The ultrasound transducer was fixed to a computer-controlled rotation stage (model 8MR190-2-4233-400 mA rotation stage and model 8SMC1-USBH controller, Standa, Vilnius, Lithuania) enabling the rotation of the transducer, with respect to the LED and the camera and, therefore, tomographic imaging. The distance between the rotation axis and the inner glass wall close to the LED was 62.5 mm. The rotation stage was connected to three-axis linear slide positioner (Time and Precision, Cheadle, UK). The transducer rotation and the imaging with the camera were synchronized using custom LabVIEW-program (LabVIEW 2010, National Instruments, Austin, TX). Tomographic measurements were done using the workstation with Microsoft Windows 7 operating system.

The driving electronics of the LED and the ultrasound transducer were synchronized by using two arbitrary waveform generators. First of them (model 33220A, Agilent, Santa Clara, CA) was used as a master trigger to control a two channel waveform generator (model AFG3102, Tektronix, Beaverton, OR). The LED driving electronics and the ultrasound driving amplifier [50 dB radiofrequency (RF) amplifier, model 240L, Electronic Navigation Industries, Rochester, NY] were connected to channels 1 and 2, respectively, of the second waveform generator. The master trigger had a pulse repetition rate of 3 kHz triggering both the LED illumination pulse and the ultrasound sonication. Channel 1 was delayed with respect to channel 2 to achieve a controlled illumination at a precise time instance \( t \) of ultrasound propagation. This way the photograph, taken by the camera, was exposed with the ultrasound field in the same phase during each light pulse. The delay was set to 66 \( \mu \text{s} \) implying ultrasound propagation distance of 98 mm.
1. Imaged target

Two different imaged targets were used. The imaged targets were printed on white A4-size laser printer paper (weight 80 g/m²).

The first target, named focusing target, was used to verify that the camera is focused on the imaged target, as well as to determine the spatial length scales of the photographs. The focusing target had a pattern composed of lines and circles of known dimension, and it was made using Adobe Illustrator CS 15.0.0 (Adobe Systems Inc., San Jose, CA). The first target was printed using HP LaserJet Pro MFP M125nw (HP Inc., Palo Alto, CA) at a resolution of 600 dots per inch (DPI).

The second target, named tomographic target, was used during the main measurements. The tomographic target consisted of a regular grid of red, green, and blue Gaussian bumps having full-width half-maximum (FWHM) of 0.3 mm. Each grid of colors was spatially interleaved. The tomographic target was generated using MATLAB R2011b (The MathWorks Inc., Natick, MA) and was printed using HP LaserJet Pro CM1415fn Color MFP (HP Inc., Palo Alto, CA) at a resolution of 600 DPI.

2. Ultrasound transducer

The ultrasound transducer in this study was an air-backed geometrically focused PZ26 (Ferroperm, Kvistgaard, Denmark) element in a polymethyl methacrylate (PMMA) housing. The element had a diameter of 45 mm and a focal length of 56 mm (f-number 1.24) and it was operated at a frequency of 1.01 MHz (wavelength \( \lambda = 1.47 \text{ mm in water} \)). The transducer was impedance matched to the driving lines electric impedance (50 \( \Omega \)) using an external LC-circuit. The transducer was operated in burst mode of 50 cycles (49.5 \( \mu \text{s} \) in duration, and 73.6 mm in spatial extent).

For comparison, the transducers’ acoustic pressure field was measured using a needle hydrophone with active diameter of 0.2 mm (Precision Acoustic, Dorchester, UK), corresponding to \( \lambda/7.4 \), and three-axis linear slide ultrasound measurement system (UMS V.1.3.5, Precision Acoustics). Measurements were done in free-field conditions inside a large water tank filled with degassed deionized water and covered with rubber mats. The acoustic waveforms were stored using a digital oscilloscope (model 6051A WaveRunner, LeCroy, Chestnut Ridge, NY) and analyzed using MATLAB R2011b (The MathWorks Inc., Natick, MA) software. Each stored waveform was averaged over 24 subsequent ultrasound pulses with sampling frequency of 100 MHz (99 points per cycle). The spatial step size of the hydrophone scans was either 0.5 mm (\( \lambda/2.9 \), used in the direction of acoustical axis, or 0.2 mm (\( \lambda/7.4 \), used in plane perpendicular to acoustical axis. The hydrophone scans were recorded with the same driving signal parameters (burst mode, driving voltage) of the ultrasound transducer as used in the SST experiment. The hydrophones sensitivity for the frequency of 1.01 MHz was determined as follows. After an acoustic power measurement, performed using a radiation force balance, the acoustic field in plane perpendicular to the acoustical axis at the focal depth was measured using the hydrophone. The hydrophone based integrated intensity was then matched to the acoustic power measurement to obtain the hydrophone sensitivity. The peak positive and peak negative pressures of the transducer on the acoustical axis were found to be 0.97 MPa and \(-0.85 \text{ MPa} \), respectively.

3. LED element and electronic circuitry

The LED used in this study was a red-light (model Asaklitt, Clas Ohlson, Dalarna, Sweden). The LED is shown in Fig. 3. Tone color of the light was cold white. The LEDs nominal power rating was 3 W. The LED, when connected to laboratory direct current (DC) source (model E3632A, Agilent, Santa Clara, CA), was fully bright when the forwarded voltage was 2.6–2.7 V, which is the typical operating voltage during continuous operation.

During the SST measurements the LED was driven in pulsed mode at a higher voltage. The schematics of the in-house built LED driving circuit are shown in Fig. 4. The circuit is based on that by Willett et al. The basic principle of this circuit is to discharge capacitors through a fast metal-oxide semiconductor field-effect transistor (MOSFET) amplifier. The capacitors (C1 and C2 in Fig. 4) are charged using laboratory DC source (E3632A) operating at \( V_{\text{LED}} = 15 \text{ V}_{\text{DC}} \), which produced sufficient light intensity without compromising the operation of the LED. MOSFET amplifier (Q1) is triggered through MOSFET driver element (U1) with trigger signal from the AFG3102 arbitrary waveform generator (J1). MOSFET driver was powered using a laboratory DC source (Model PP5007, Farnell, Leeds, UK) operating at \( V_{\text{f}} = 12 \text{ V}_{\text{DC}} \). The diode D1 was placed to protect the LED (LD1) from possible reverse currents. The nominal pulse duration was 70 ns, which was observed to produce the shortest pulse of light with the current circuitry.

The duration of the LED pulse was measured using silicon PIN photodiode (model S1223, Hamamatsu Photonics, Hamahatsu, Japan) connected to a digital oscilloscope (LeCroy 6051A WaveRunner) through an amplifying circuitry. These measurements were made using the same...
setup as in the actual SST experiments, except that the digital camera was replaced with the photodiode and the ultrasound transducer was not operated. The photodiode was positioned 10 mm from the water-filled tank glass wall. The measured photodiode waveforms when the glass tank was either in place or absent are shown in Fig. 5.

The light pulse rise time (defined as the time for the signal to rise from 10% to 90% of the maximum value) was found to be $t_{\text{RISE}} = 111$ ns when the light was transmitted through the water-filled tank. The pulse fall time (defined as the time for the signal to fall from 90% to 10% of the maximum value) was found to be $t_{\text{FALL}} = 190$ ns. FWHM time was $t_{\text{FWHM}} = 181$ ns. The corresponding values, when the water-filled tank was absent and the pulse traveled from the LED through air to the photodiode, were $t_{\text{RISE}} = 85$ ns, $t_{\text{FALL}} = 176$ ns, and $t_{\text{FWHM}} = 196$ ns. The intensity reaching the photodiode was attenuated by 6.6 dB when the tank was placed between the LED and the photodiode.

C. Measurement and analysis protocols

After assembling the measurement setup, the focusing target was placed behind the measurement tank. Auto-focus feature of the camera was used to focus on the focusing target and an image was captured in a lit room. Following that, focusing target was replaced with the tomographic target. After dimming the laboratory, a photograph of the tomographic target was captured without ultrasound sonication. Next the ultrasound was switched on, and tomographic measurement was performed with 180 photographs taken at 1 degree intervals.

After the measurements, the photograph of the focusing target was used to determine spatial length scale of the images in the plane of the focusing target. The length scale at the focusing target was then used to estimate the length scale at the rotation axis. The process is depicted in Fig. 6.

According to Snell’s law, the refractive angles on interfaces normal to the water tank follow

$$n_{\text{air}} \sin \theta_1 = n_{\text{glass}} \sin \theta_2 = n_{\text{water}} \sin \theta_3 = n_{\text{glass}} \sin \theta_4,$$

(18)

where refractive indices $n_{\text{air}} = 1$, $n_{\text{glass}} = 1.5$, $n_{\text{water}} = 1.34$ have been assumed, and $\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$ are the incidence angles from camera to air, air to glass tank, glass to water, and water to glass, respectively. It follows, that the displacement of light rays from optical axis behind the water tank (i.e., at the focusing target) is

$$\Delta h = w_{\text{air}} \tan \theta_1 + w_{\text{glass}} \tan \theta_2 + w_{\text{water}} \tan \theta_3 + w_{\text{glass}} \tan \theta_4,$$

(19)

and the displacement of light ray at the rotation axis is

$$\Delta h' = w_{\text{air}} \tan \theta_1 + w_{\text{glass}} \tan \theta_2 + (w_{\text{water}} - w_{\text{rot}}) \tan \theta_3,$$

(20)

where $w_{\text{air}} = 412$ mm, $w_{\text{glass}} = 5$ mm, $w_{\text{water}} = 286$ mm, and $w_{\text{rot}} = 62.5$ mm are distances from camera to water tank, thickness of glass, the inner thickness of water tank, and distance of rotation axis to inner wall of the water tank, respectively. It was determined, from the known dimensional scale.
present in the focusing target that $\Delta h = 53.6 \mu m/pixel$ at the focusing plane. From Eqs. (18)–(20) it follows that $\Delta h' = 49.4 \mu m/pixel$ at the rotation axis, which is used to define the spatial scale in all of the SST analysis.

The photographs of the tomographic target were cropped and centered around the rotation axis, which was visible in the uncropped photographs. Horn-Schunck algorithm [Eq. (9)], with photograph of the tomographic target without ultrasound sonication, and photographs at the 180 rotation directions during sonication, was used to obtain $v(r, \theta, t)$ by using MATLAB R2011b (The MathWorks Inc., Natick, MA). The iradon function of MATLAB was then used to perform inverse Radon transform using filtered back-projection, with Hann filtering window, to obtain a 3D representation of $\partial p/\partial y$. Approximation equation (17) was then used to obtain the 3D pressure field. During the computations the constant multipliers were neglected, and thus the 3D pressure field was obtained in arbitrary units.

III. RESULTS

Recorded grayscale photographs of the tomography target without and with ultrasound sonication, and their difference, are shown in Fig. 7. The photographs have been cropped to a $1025 \times 2049$ pixels ($50.6 \times 101.2$ mm at the rotation axis) within the region of effective illumination and centered on the axis of rotation. Figure 7 also depicts close-up of the photograph, showing the dotted pattern of the tomography target in a $129 \times 257$ pixel region ($6.4 \times 12.7$ mm at the rotation axis). Qualitative inspection of the photographs shows no clearly visible perturbation, although the difference image shows horizontal banding (especially in the close-up) suggesting existence of banded perturbation field, such as caused by the ultrasound. Figure 8 shows the displacement (pressure gradient projection), as computed with the Horn-Schunck algorithm [Eq. (9)], obtained from photographs shown in Fig. 7. Figure 8 encodes the 2D displacement vectors into color according to the visible color wheel. Perturbation resembling an ultrasound field is clearly observable.

Figure 9 shows planar slices of the instantaneous pressure field of the focused ultrasound transducer as obtained with SST, and the corresponding hydrophone scans. The SST pressure has been normalized to unit peak positive amplitude based on the 3D pressure field. Qualitatively the SST and the hydrophone scans look similar to each other.

Figure 10 shows the instantaneous normalized pressure on the acoustical axis of the transducer as obtained with SST, and the corresponding hydrophone scan. The phase of the ultrasound on the acoustical axis obtained with SST appears to be well captured with respect to hydrophone data. The SST pressure seems to fall mostly between the peak positive and negative pressure contour obtained with hydrophone shown in light gray. Figure 11 shows the instantaneous normalized pressure profile on two axes perpendicular to the acoustical axis for the SST and the hydrophone. Based on the pressure profiles, the SST is capable of producing qualitatively similar information as the hydrophone scans; however, the accuracy and calibration of the technique requires further work. Quantitatively, the FWHM of peak positive instantaneous pressure in SST at $y = 0$ mm plane in the $x$-direction is $3.3$ mm, and in the $z$-direction $3.1$ mm. The

![FIG. 7. Grayscale photograph of the tomography target without ultrasound sonication (left), with ultrasound sonication (middle), and the difference image (right). Bottom row shows a close-up of the white rectangle shown in the top row. The ultrasound transducer is located in the top of the images and sonicating downward.](image)

![FIG. 8. (Color online) Color encoded displacement (pressure gradient projection) computed from photographs shown in Fig. 7. The right image is a close-up of the black rectangle seen on the left. The color wheel in the inset in the left image encodes colors into 2D displacement vector. The ultrasound transducer is located in the top of the images and sonicating downward.](image)
corresponding values in the hydrophone scans were 2.7 mm and 2.5 mm.

IV. DISCUSSION

In this work, synthetic schlieren, or BOS, imaging was extended into tomographic imaging of pulsed ultrasound fields. The tomographic imaging was performed using a LED light source and a digital camera. The approach was tested with a focused ultrasound transducer, operating at 1.01 MHz with peak pressure amplitude of 0.97 MPa, i.e., in medically relevant parameter scale. The pressure field obtained with the approach was compared to hydrophone scans. While the fields were qualitatively comparable, it was observed, that the SST resulted in broadening of the pressure field in plane perpendicular to acoustical axis.

SST could offer significant reduction in measurement time in comparison with traditional hydrophone measurements. Hydrophone scan covering a single 2D plane with 41 × 41 points at 0.5 mm spatial resolution took approximately 2 h and 10 min. The SST measurement (180 rotations in one degree increments) took approximately 50 min. The measurement time could be shortened by using a more tailored camera software and higher-speed communication bus (if available). In current setup, the most of the measurement time was spent in communication between the digital camera and gphoto2 software, and in image transfer from the camera to computer through USB 2.0. It was estimated that ~40 min of the total time was consumed in these processes. The analysis of the photographs, and the estimation of the 3D pressure field, took approximately 6 h and 10 min to compute, of which 6 h was spent on computing 180 displacement fields for each of the rotation angles (about 2 min each) using the Horn-Schunck algorithm. The computation was done on a workstation equipped with Xeon E5649 (Intel Corporation, Santa Clara, CA) processor operating at 2.53 GHz using MATLAB R2011b (The MathWorks Inc., Natick, MA). The analysis was not parallelized, and it should be noted that evaluation of 180 Horn-Schunck algorithms can be done independently of each other, implying that a significant reduction in computational time could be achieved. The size of the 3D SST volume was 724 × 724 × 2049 voxels with spatial resolution of 49.4 μm.

The light source used in this work was an LED obtained from an ordinary flashlight. The measured light pulse width...
This implies that the images obtained in this study were averaged over half wavelength distance. This is typically an acceptable amount when using, e.g., needle hydrophones. Spatial averaging caused by the finite camera aperture could be reduced by imaging using larger f-number at the cost of longer imaging time due to reduced light exposure. No calibrations for any lens aberrations were performed in this work. The authors acknowledge that the current approach used to determine spatial length scale at the rotation axis based on photograph of camera focusing target is not ideal and is likely to cause some amount of error in the current setup.

The Horn-Schunck optical flow algorithm used in this work was chosen due to its good performance with respect to noise. This performance is achieved by weighting the algorithm (via Laplacian based regularization) toward displacement fields that are spatially smooth, which are to be expected when imaging ultrasound fields. However, in this work the regularization parameter was not thoroughly optimized and it was chosen based on visual inspection of the displacement fields. The approach could potentially be made more suitable for estimation of ultrasound fields by incorporating more complex prior information, instead of just a Laplacian based squared Euclidean 2-norm, as well as by incorporating more thorough image noise characterization into the model. It is also possible, that optimizing the printed tomographic target could result in improved results, as found in Ref. 16.

The Horn-Schunck optical flow algorithm used in this work was an ordinary consumer level camera equipped with ordinary lenses. No thorough optimization on finding the best possible imaging parameters was attempted in this work. While increasing the distance between the camera and the measurement tank increases the displacement of light rays, the angular resolution is decreased. This implies that there exists an optimal distance for a given lens and camera combination with respect to spatial resolution requirements set by the ultrasound transducers’ operating frequency. The effective aperture diameter of the camera, with the parameters used, is 9.8 mm. Using an approach similar to Sec. II C it was estimated that the width of a cone of light originating from a point in the imaged target as it passes the ultrasound rotation axis is 0.68–0.78 mm corresponding to 47%–53% of wavelength of ultrasound. This implies that the images obtained in this study were averaged over half wavelength distance. This is typically an

for the current LED was $\sim$181 ns. Neumann and Ermert$^{43}$ give a value for ultrasound phase shift during optical pulse propagation as a product of ultrasound frequency and the light pulse width. This indicates that the ultrasound wave was spatially shifted and thus, averaged, by $\sim$18% of the acoustic wavelength during the propagation of light pulse. Based on presented data this value is acceptable as it produced results that are comparable to hydrophone measurements. For the lower acoustic frequencies, the pulse width could be increased resulting in shorter camera exposure times. The LED used in this study was not chosen due to its optimal response times and other LEDs may offer similar or improved performance required for measuring higher ultrasonic frequencies.

The camera used in this work was chosen based on visual inspection of the displacement for any lens aberrations were performed in this work. The authors acknowledge that the current approach based on photograph of camera focusing target is not ideal and is likely to cause some amount of error in the current setup. This work has been supported by the Academy of Finland Project No. 286247.

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