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We present mathematical models for temporally and spectrally partially coherent pulse trains with Laguerre-Gaussian and Hermite-Gaussian Schell-model statistics as extensions of the standard Gaussian Schell model for pulse trains. We derive propagation formulas of both classes of pulsed fields in linearly dispersive media and in temporal optical systems. It is found that, in general, both types of fields exhibit time-domain self-splitting upon propagation. The Laguerre-Gaussian model leads to multiply peaked pulses, while the Hermite-Gaussian model leads to doubly peaked pulses, in the temporal far field (in dispersive media) or at the Fourier plane of a temporal system. In both model fields the character of the self-splitting phenomenon depends both on the degree of temporal and spectral coherence and on the power spectrum of the field.

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I. INTRODUCTION

Various partially coherent model sources have gained a great deal of attention in the past few years [1], which is largely due to the extraordinary propagation properties of the fields they generate. Partially coherent fields can cause several different kinds of modifications to the spatial intensity distribution, such as self-focusing and far-field flat topping, among many others [2–16]. However, they are not the only extraordinary phenomena caused by partial coherence; some correlation functions can decrease scintillation in atmospheric turbulence [17], while others increase the resolving power of imaging systems [18,19]. Many of these sources have been experimentally generated, and their properties are now rather well known [20–27]. A detailed review on generation and propagation of various partially coherent beams can be found in [28] and the references therein.

However, all of the aforementioned investigations are confined to wide-sense statistically stationary light sources or beams in the spatial domain. In addition to spatial domain effects, optical pulses with partial spectral or temporal coherence have recently attracted researchers’ widespread attention, for their possible applications in optical telecommunication, imaging, and fiber optics [29–41]. Temporally partially coherent pulse trains are generated by many real sources, such as free-electron and excimer lasers, supercontinuum in microstructured fibers, or random lasers [42], although practical schemes for controlling or modulating the temporal (or spectral) coherence are scarce [43].

In the majority of theoretical studies concerning partial temporal coherence, the correlations have been chosen to be of the Gaussian Schell-model (GSM) form. That is, the mean temporal intensity and the spectral density of the pulse train are both Gaussian functions, as are the two-time degree of temporal coherence and the two-frequency degree of spectral coherence, which both depend on the appropriate coordinate difference only. Considering some of the more exotic correlation functions, one can readily produce similar effects in time domain as one has in the spatial domain, including acceleration of the intensity peak, pulse self-splitting, and pulse flat topping [44–47]. Out of these, temporal self-splitting is of potential interest in optical data transmission, since it may open up the possibility to encrypt data in a way that unravels itself upon propagation. This would require flexible models of self-splitting pulse trains and methods to gain control over the self-splitting properties, which have been elusive until now.

In the present paper, we first describe genuine pulse representations in both temporal and spectral domains in Sec. II, as counterparts of such representations in the spatial domain. We also briefly recall the GSM pulses before proceeding to Sec. III, where we introduce two classes of self-splitting pulse models, the Laguerre-Gaussian correlated Schell-model (LGCSM) pulses in Sec. III A and Hermite-Gaussian correlated Schell-model (HGCSM) pulses in Sec. III B. The propagation formulas for these pulses in temporal optical systems are derived in Sec. IV, with particular attention to temporal self-splitting upon propagation through linearly dispersive media. Finally, some conclusions along with remarks on practical realization and potential applications of the models are provided in Sec. V.

II. PULSE REPRESENTATIONS

Plane-wave pulse trains with arbitrary coherence properties can be described in the time domain by means of the two-time mutual coherence function (MCF), which is defined as the ensemble average

\[
\Gamma(t_1,t_2) = \langle E^\dagger(t_1)E(t_2) \rangle
\]

\[
= \lim_{n\to\infty} \frac{1}{N} \sum_{n=1}^{N} E_n^\dagger(t_1)E_n(t_2) \quad (1)
\]
over all possible temporal pulse realizations $E_q(t)$. In analogy with space-domain correlation functions \cite{48}, the MCF can always be expressed in the genuine form

$$\Gamma(t_1, t_2) = \int_{-\infty}^{\infty} p(v) h^*(t_1, v) h(t_2, v) dv,$$  \hspace{1cm} (2)

where $p(v)$ is a non-negative weight function and $h(t, v)$ is an arbitrary and possibly complex-valued kernel. We will take the weight function as being normalized, such that $\int_{-\infty}^{\infty} p(v) dv = 1$. The genuine representation is one of the experimentally realizable ways of producing partially coherent sources. Simply put, the weight function describes how the kernel functions (electric fields) should be distributed to produce the desired correlation. If a correlation function does not have a genuine representation, then it is not physically realizable.

Let us assume that the time-domain kernel has the specific (Fourier-type) form

$$h(t, v) = f(t) \exp(-i tv) = a(t) \exp[-i(v + \omega_0) t],$$  \hspace{1cm} (3)

where the envelope $f(t) = a(t) \exp(-i(v + \omega_0) t)$ of the temporal field with a carrier frequency $\omega_0$ is used to write the second equality. In this case, the MCF of the pulse train is of the Schell-model form \cite{49}

$$\Gamma(t_1, t_2) = f^*(t_1) f(t_2) g(\Delta t)$$

$$= a^*(t_1) a(t_2) g(\Delta t) \exp(-i \omega_0 \Delta t),$$  \hspace{1cm} (4)

where $\Delta t = t_2 - t_1$ and

$$g(\Delta t) = \int_{-\infty}^{\infty} p(v) \exp(-i \Delta tv) dv$$  \hspace{1cm} (5)

satisfies the condition $|g(0)| = 1$. Under these assumptions, the mean temporal intensity of the pulse train is

$$I(t) = \Gamma(t, t) = |f(t)|^2 = |a(t)|^2$$  \hspace{1cm} (6)

and its complex degree of temporal coherence can be written as

$$\gamma(t_1, t_2) = \frac{\Gamma(t_1, t_2)}{\sqrt{I(t_1)I(t_2)}}$$

$$= g(\Delta t) \exp[i[\phi(t_2) - \phi(t_1)] - i \omega_0 \Delta t],$$  \hspace{1cm} (7)

where $\phi(t) = \arg a(t)$.

In addition to considering fields that are of the Schell-model form in the time domain, we can also consider their spectral-domain counterparts. The two-frequency cross-spectral density function (CSD) $W(\omega_1, \omega_2)$ is obtained from the MCF using the generalized Wiener-Khintchine theorem

$$W(\omega_1, \omega_2) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Gamma(t_1, t_2)$$

$$\times \exp[-i(\omega_1 t_1 - \omega_2 t_2)] dt_1 dt_2.$$  \hspace{1cm} (8)

On inserting from Eq. (4) it follows that the CSD has a genuine representation

$$W(\omega_1, \omega_2) = \int_{-\infty}^{\infty} p(v) H^*(\omega_1, v) H(\omega_2, v) dv,$$  \hspace{1cm} (9)

where the frequency-domain kernel $H(\omega, v)$ forms a Fourier transform pair with its temporal counterpart,

$$H(\omega, v) = \frac{1}{2\pi} \int_{-\infty}^{\infty} h(t, v) \exp(i \omega t) dt.$$  \hspace{1cm} (10)

Let us assume that the spectral-domain kernel can be written as

$$H(\omega, v) = \frac{1}{\sqrt{2\pi}} F(\sigma) \exp(i \omega v),$$  \hspace{1cm} (11)

where $\sigma = \omega - \omega_0$, with $v$ now being interpreted as a temporal rather than a spectral variable. Consequently, the CSD takes the Schell-model form

$$W(\omega_1, \omega_2) = F^*(\sigma_1) F(\sigma_2) G(\Delta \omega),$$  \hspace{1cm} (12)

where $\Delta \omega = \omega_2 - \omega_1 = \sigma_2 - \sigma_1$ and we have defined

$$G(\Delta \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} p(v) \exp(i \Delta \omega v) dv$$  \hspace{1cm} (13)

so that $|G(0)| = 1$. The mean spectrum of the pulse train is now

$$S(\omega) = W(\omega, \omega) = |F(\sigma)|^2,$$  \hspace{1cm} (14)

and its complex degree of spectral coherence is

$$\mu(\omega_1, \omega_2) = \frac{W(\omega_1, \omega_2)}{\sqrt{S(\omega_1)S(\omega_2)}}$$

$$= G(\Delta \omega) \exp[i[\Phi(\sigma_2) - \Phi(\sigma_1)]],$$  \hspace{1cm} (15)

with $\Phi(\sigma) = \arg F(\sigma)$. Thus, if we know the degree of temporal or spectral coherence of a Schell-model pulse train, as well as the temporal intensity or spectrum, we can find the genuine representation. It is necessary to ascertain that a genuine form exists for the fields we consider here, or otherwise they would not correspond to any real fields.

Considering the temporal domain, we get the weight function by inversion of Eq. (5), as in

$$p(v) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(\Delta t) \exp(i v \Delta t) d\Delta t,$$  \hspace{1cm} (16)

the amplitude and phase of $a(t)$ being determined by Eqs. (6) and (7), respectively. Correspondingly, in the frequency domain, the inversion of Eq. (13) gives

$$p(v) = \int_{-\infty}^{\infty} G(\Delta \omega) \exp(-i v \Delta \omega) d\Delta \omega,$$  \hspace{1cm} (17)

while the amplitude and phase of $F(\sigma)$ are determined by Eqs. (14) and (15), respectively.

As a fundamental example, we can consider the well-known GSM pulses \cite{30}, for which we can write the CSD as

$$W(\sigma_1, \sigma_2) = \sqrt{S(\sigma_1)S(\sigma_2)} \mu(\Delta \omega),$$  \hspace{1cm} (18)

where the spectral density is

$$S(\sigma) = S_0 \exp\left(-\frac{2\sigma^2}{\Omega_0^2}\right)$$  \hspace{1cm} (19)

and the complex degree of spectral coherence is given by

$$\mu(\Delta \omega) = G(\Delta \omega) = \exp\left(-\frac{\Delta \sigma^2}{2\Omega_0^2}\right).$$  \hspace{1cm} (20)
The MCF of GSM pulses can also be expressed as
\[ \Gamma(t_1,t_2) = \sqrt{I(t_1)I(t_2)} \gamma(\Delta t), \] (21)
where we have the temporal intensity
\[ I(t) = I_0 \exp \left( -\frac{2t^2}{T^2_0} \right) \] (22)
and the complex degree of temporal coherence is
\[ \gamma(\Delta t) = \exp \left( -\frac{\Delta t^2}{2T^2_c} \right) \exp(-i\omega_0 \Delta t). \] (23)

Additionally, the pulse duration \( T_0 \), coherence time \( T_c \), spectral bandwidth \( \Omega_0 \), and the coherence width \( \Omega_c \) are related by
\[ \frac{\Omega_0^2}{4} = \frac{1}{T^2_0} + \frac{1}{T^2_c}, \] (24)
\[ \Omega_c = \frac{T_c}{T_0} \Omega_0. \] (25)

It will also be convenient to introduce an abbreviation
\[ \alpha = \left( 1 + \frac{T^2_0}{T^2_c} \right)^{-1/2} = \left( 1 + \frac{\Omega^2_0}{\Omega^2_c} \right)^{-1/2}. \] (26)

Letting \( \Omega_c \rightarrow 0 \) (or \( \alpha \rightarrow 0 \)) means full spectral incoherence. Then, for any value of spectral bandwidth \( \Omega_0 \), the pulse duration \( T_0 \rightarrow \infty \), and the field becomes temporally stationary. The opposite limit \( \Omega_c \rightarrow \infty \) (or \( \alpha \rightarrow 1 \)) means full spectral coherence, which also implies full temporal coherence \( (T_c \rightarrow \infty) \) and the usual relation \( T_0 = 2/\Omega_0 \) applies between the temporal duration and the spectral bandwidth of coherent Gaussian pulses. Partially coherent pulse trains are obtained if \( \Omega_0 T_0 > 2 \), and if \( T_0 \) and \( \Omega_0 \) are known, the coherence time of such pulse trains is given, according to Eq. (25), by
\[ T_c = T_0 \left[ \frac{\Omega_0 T_0}{2} \right]^{-1/2}. \] (27)

Hence the coherence time \( T_c \) is larger than zero for any bandlimited train of pulses with finite duration.

From the expressions given above, we immediately find that the weight function and the kernels in both time and frequency domains are all of Gaussian form for GSM pulses.

### III. SELF-SPLITTING PULSES

We proceed to extend the GSM pulses by introducing classes of model pulses that will both exhibit temporal self-splitting upon propagation. We first consider LGCSM pulses in Sec. III A and HGCSM pulses in Sec. III B.

#### A. Laguerre-Gaussian correlated Schell-model pulses

Let us describe the time-domain weight function \( p(v) \) as
\[ p(v) = \frac{1}{2\pi n! \sqrt{T^2_0}} H^2_n(v) \exp(-v^2), \] (28)
where \( H_n \) is a Hermite polynomial of order \( n \). This is now clearly different from the basic GSM case, seeing that the weight function consists of a Gaussian function and a Hermite polynomial, and it will lead to a temporal coherence function that is modulated by a Laguerre polynomial. We define these

![FIG. 1. Degree of coherence \( g(\Delta t) \) for the LGCSM pulses \( \Delta t/T_c \) and for different values of Laguerre order \( n \). Dashed red: \( n = 1 \). Dash-dotted green: \( n = 2 \). Dotted blue: \( n = 3 \).

By inserting from Eqs. (28) and (29) into Eq. (2), we get
\[ \Gamma(t_1,t_2) = \frac{1}{2^n n! \sqrt{\pi}} \exp(-i\omega_0 \Delta t) \exp \left( -\frac{t^2 + \tilde{t}^2}{T^2_0} \right) \times \int_{-\infty}^{\infty} \frac{\Delta t}{T_c} H^2_n(v) \exp(-v^2) \exp \left( -i\sqrt{\frac{\Delta t}{T_c}} v \right) dv. \] (30)

The integral can be evaluated with the aid of Eq. (A3), resulting in
\[ \Gamma(t_1,t_2) = L_n \left( \frac{\Delta t^2}{T^2_c} \right) \exp \left( -\frac{t^2 + \tilde{t}^2}{T^2_0} \right) \times \exp \left( -\frac{\Delta t^2}{2T^2_c} \right) \exp(-i\omega_0 \Delta t), \] (31)
where \( L_n \) is a Laguerre polynomial of order \( n \). The mean intensity of the pulse is a Gaussian function
\[ I(t) = \exp \left( -\frac{2t^2}{T^2_0} \right), \] (32)
and using Eq. (7), we find that the complex degree of temporal coherence is of the form \( \gamma(\Delta t) = g(\Delta t) \exp(-i\omega_0 \Delta t) \) with
\[ g(\Delta t) = L_n \left( \frac{\Delta t^2}{2T^2_c} \right) \exp \left( -\frac{\Delta t^2}{T^2_c} \right). \] (33)

Expressions (31)–(33) define the class of LGCSM pulses, which reduce to the GSM case described by Eqs. (21)–(23), if \( n = 0 \). Figure 1 illustrates the degree of temporal coherence \( g(\Delta t) \) for LGCSM pulses for some different values of \( n \).

Let us next consider the spectral properties of the LGCSM pulses. Inserting from Eq. (29) into Eq. (10) we obtain the spectral-domain kernel
\[ H(\omega,v) = \frac{T_0}{2\sqrt{\pi}} \exp \left[ -\frac{T^2_0}{4} \left( \sigma - \frac{\sqrt{2}v}{T_c} \right)^2 \right]. \] (34)
Further, on substituting from Eqs. (28) and (34) into Eq. (9), and using Eqs. (24) and (25) to introduce \( \Omega_c \) and \( \Omega_0 \), we arrive at an integral expression that can be evaluated with the aid of Eq. (A4), obtaining

\[
W(\omega_1, \omega_2) = \frac{T_0}{2\pi 2^n n! \Omega_0} \exp \left( -\frac{\Delta \omega^2}{2\Omega_0^2} \right) \sum_{k=0}^{\infty} \frac{2^k k! (n^2)}{(1 + \Omega_0^2 / \Omega_0^2)^{n-k}} H_{2(n-k)} \left( \frac{\sqrt{2}\sigma_0^2}{\Omega_0} \right).
\]

where the two exponential terms in front of the integral were also rearranged. The spectral density is now

\[
S(\omega) = \frac{T_0}{2\pi 2^n n! \Omega_0} \exp \left( -\frac{2\sigma^2}{\Omega_0^2} \right) \sum_{k=0}^{\infty} \frac{2^k k! (n^2)}{(1 + \Omega_0^2 / \Omega_0^2)^{n-k}} H_{2(n-k)} \left( \frac{\sqrt{2}\sigma_0^2}{\Omega_0} \right).
\]

In the limit of complete incoherence \( \Omega_c \to 0 \), which implies a temporally stationary field, Eq. (A5) can be used to simplify Eq. (36) to the form

\[
S(\omega) = \frac{T_0}{2\pi 2^n n! \Omega_0} \exp \left( -\frac{2\sigma^2}{\Omega_0^2} \right) \left( \frac{\sqrt{2}\sigma_0^2}{\Omega_0} \right).
\]

showing that a Gaussian spectrum is modulated by a squared Hermite polynomial of order \( n \).

Figure 2 illustrates some spectral density profiles of LGCSM sources with selected values of order index \( n \) and the ratio \( \Omega_c / \Omega_0 \). With \( n = 0 \) the spectrum is Gaussian, but with \( n > 0 \) it is in general split into several lobes. Such splitting vanishes in the coherent limit \( \Omega_c \to \infty \), but becomes distinct when the degree of coherence is reduced. In the spectrally incoherent (stationary) limit, \( \Omega_c / \Omega_0 \to 0 \), we obtain \( n+1 \) lobes separated by zeros.

**B. Hermite-Gaussian correlated Schell-model pulses**

Let us next take the mean intensity as being of the Gaussian form of Eq. (32), but assume that the complex degree of temporal coherence is—instead of Eq. (33)—determined by the function

\[
g(\Delta t) = \frac{H_{2n}(\Delta t / \sqrt{2}T_c)}{H_{2n}(0)} \exp \left( -\frac{\Delta t^2}{2T_c^2} \right).
\]

These choices define the class of HGCSM pulse trains, and we can again see that the degree of temporal coherence reduces to that of conventional GSM pulses for \( n = 0 \). In Fig. 3 we illustrate the function \( g(\Delta t) \) for HGCSM pulses with some different values of \( n \).

It follows from Eq. (16) that the weight function of the genuine representation of HGCSM pulse trains is given by

\[
p(v) = \frac{1}{2\pi H_{2n}(0)} \exp \left( -\frac{1}{2} T_c^2 v^2 \right) \times \int_{-\infty}^{\infty} H_{2n} \left( \frac{\Delta t}{\sqrt{2}T_c} \right) \times \exp \left[ -\left( \frac{\Delta t}{\sqrt{2}T_c} - i \frac{T_c v}{\sqrt{2}} \right)^2 \right] d\Delta t.
\]

**FIG. 2.** Normalized spectral density \( S(\omega)/S_0 \) with \( S_0 = T_0/2\pi \Omega_0 \) when (a) \( n = 1 \) and (b) \( n = 4 \). Solid black: \( \Omega_c / \Omega_0 = 10 \). Dashed red: \( \Omega_c / \Omega_0 = 1 \). Dash-dotted green: \( \Omega_c / \Omega_0 = 0.5 \). Dotted blue: \( \Omega_c / \Omega_0 = 0.1 \).

**FIG. 3.** Same as Fig. 1 but for HGCSM pulses.
where $1/H_{2c}(0) = (-1)^n 2^n/(2n - 1)$!! The integral can be evaluated using Eq. (A6), which gives

$$p(v) = \frac{1}{\sqrt{\pi}} \frac{2^n}{(2n - 1)!!} \left( \frac{T_c}{\sqrt{2}} \right)^{2n+1} v^{2n} \exp\left(-\frac{1}{2} T_c^2 v^2\right).$$

(40)

Since we assume a Gaussian intensity distribution given by Eq. (32), using Eq. (3) gives the temporal-domain kernel as

$$h(t,v) = \exp\left(-\frac{v^2}{T_c^2}\right) \exp\left[-i(v + \omega_0)t\right],$$

(41)

and according to Eq. (10), its spectral counterpart is

$$H(\omega,v) = \frac{T_0}{2\sqrt{\pi}} \exp\left[-\frac{1}{4} T_0^2 (\sigma - v)^2\right].$$

(42)

Substituting into Eq. (9), and employing Eqs. (24), (25), and (A7), we get the CSD as

$$W(\omega_1,\omega_2) = \frac{(-1)^n T_0}{2\pi} \frac{2^n \alpha^{2n}}{\Omega_0 (2n - 1)!!} \times \exp\left(-\frac{\alpha_1^2 + \alpha_2^2}{\Omega_0^2}\right) \times H_\alpha\left[\frac{i}{\sqrt{2\Omega_c}} (\alpha_1 + \alpha_2)\right].$$

(43)

and the spectral density is then given by

$$S(\omega) = \frac{(-1)^n T_0}{2\pi} \frac{2^n \alpha^{2n}}{\Omega_0 (2n - 1)!!} \times \exp\left(-\frac{2\sigma^2}{\Omega_c^2}\right) H_\alpha\left[\frac{i\sqrt{2}\sigma}{\Omega_c}\right].$$

(44)

Clearly, when $n = 0$, GSM pulse trains are again obtained.

We plot in Fig. 4 the normalized spectral density for the HGCSM pulses for some values of $n$ and the ratio $\Omega_c/\Omega_0$. The trends are similar to the case of LGCSM pulses: splitting becomes increasingly prominent for all $n > 1$ when the degree of coherence is reduced. However, now the spectra are split in only two lobes. In the stationary limit $\Omega_c/\Omega_0 \rightarrow 0$ these lobes become separated, and the spectral distance between their peaks increases with $\sigma$.

IV. PROPAGATION THROUGH TEMPORAL OPTICAL SYSTEMS

Propagation of the mutual coherence function through dispersive media can be investigated by using the generalized Collins formula in the temporal domain [31,50]

$$\Gamma(t_1,t_2,z) = \frac{\omega_0}{2\pi B} \exp\left[-\frac{i\omega_0 D}{2B} (t_1^2 - t_2^2)\right] \times \int \int \int_\infty^{-\infty} \Gamma_0(t_1,t_2) \exp\left[-\frac{i\omega_0 A}{2B} (t_1^2 - t_2^2)\right] \times \exp\left[\frac{i\omega_0 C}{B} (t_1 t_1 - t_2 t_2)\right] dt_1 dt_2.$$

(45)

where $A$, $B$, $C$, and $D$ are the elements of an arbitrary temporal transfer matrix and $\Gamma_0(t_1,t_2)$ is the MCF at $z = 0$.

The generalized Collins formula works only up to the second-order dispersion, since higher-order dispersion effects produce temporal aberrations, for which no temporal transfer matrices exist. Here we take the time coordinate as being measured in the reference frame moving at the group velocity of the pulse, and for mathematical convenience we introduce the following average and difference coordinates:

$$\tau = \frac{1}{2} (t_1 + t_2), \quad \Delta \tau = t_2 - t_1,$$

$$t = \frac{1}{2} (t_1 + t_2), \quad \Delta t = t_2 - t_1.$$

(46)

A. Propagation of Laguerre-Gaussian correlated Schell-model pulses

Let us start by considering LGCSM pulses. On substituting from Eq. (31) into Eq. (45) and changing coordinates according to Eq. (46), we obtain the following propagation formula:

$$\Gamma(t,\Delta t,z) = \frac{\omega_0}{2\pi B} \exp\left[\frac{iD\omega_0}{B} \Delta t\right] \times \int \int \int_\infty^{-\infty} \exp\left[-\frac{2\pi^2}{\Omega_0^2}\right] \exp\left[\frac{i\omega_0}{B} (A\Delta \tau - \Delta t)\right] d\tau \times \exp\left[-\frac{\Delta \tau^2}{2\Omega_c^2} - \frac{\Delta \tau^2}{2\Omega_c^2} - \frac{i\omega_0}{B} \Delta t\right] L_n\left(\frac{\Delta \tau^2}{T_c^2}\right) d\Delta t.$$

(47)

First integrating over $\tau$ and expanding the Laguerre function with Eq. (A8), we can express Eq. (47) in a form that allows...
evaluation of the remaining integral with Eq. (A7), and the propagated MCF then becomes
\[
\Gamma(t_1, t_2, z) = \frac{\omega_0 T_0}{2 \sqrt{2} B} \exp \left[ -\frac{i \omega_0 D}{2 B} \left( t_1^2 - t_2^2 \right) \right] 
\times \exp \left[ -\frac{T_0^2 \omega_0^2}{8 B^2} (t_1 - t_2)^2 \right] \exp \left[ -\frac{d^2(t_1, t_2)}{4 c} \right] 
\times \sum_{q=0}^{n} \left( \begin{array}{c} n \\ q \end{array} \right) \frac{1}{q! (2 T_c)^{\frac{3}{2} q^2} c^{q + 1/2}} \left( \frac{\sqrt{2} t}{T(z)} \right)^{2 q} H_{2q},
\]
(48)
where we have employed the shorthand notations
\[
c = \frac{1}{2 T_0^2} + \frac{1}{2 T_c^2} + \frac{T_0^2}{8} \left( \frac{A \omega_0}{B} \right)^2,
\]
(49)
and \(d = d(t_1, t_2) = \frac{\omega_0}{2B} (t_2 + t_1) + iT_0^2 \frac{A \omega_0}{4B^2} (t_2 - t_1).\)
(50)

Expression (48) gives the MCF at the output plane of an arbitrary temporal optical system, but here our main interest is to study the evolution of the temporal intensity profile as the pulse propagates in a linearly dispersive medium, in which the pulse propagates without a change of spatially partially coherent fields \[51,52\], but not (to our knowledge) extensively in the temporal domain. A typical characteristic of far-zone propagation, well known in the spatial domain, is that the field propagates without a change in its functional form but only in scale. This characteristic is evident for the temporal LGCSM fields illustrated in Fig. 5 at distances \(z \gg z_T\). Such a behavior is natural in view of the space-time duality in optics \[53,54\], which has been examined also in the domain of optics of coherent ultrashort pulses \[55\]. The numerical results in Fig. 5 evidence the applicability of space-time duality and the associated far-zone propagation criteria to pulsed optical fields with partial coherence properties.

In Fig. 5, we show how the intensity \(I(t, z)\) of LGCSM pulses with different orders \(n\) evolve upon propagation through a dispersive temporal system. We have chosen the simulation parameters as \(T_0 = 30\) ps and \(\beta_2 = 50\) ps\(^2\) km\(^{-1}\), which are close to typical values in optical telecommunication. Here we assume \(T_c = 10\) ps, and the plots extend to the far zone (temporal Fraunhofer region), where \(z \gg z_T\) with \(z_T = 2.85\) km with the chosen parameters. It is clear from Fig. 5 that the order \(n\) plays an important role in the pulse-splitting effect in the case of LGCSM pulses. For bare GSM pulses with \(n = 0\), there is no pulse-splitting effect during propagation, as is well known. However, higher-order LGCSM pulses exhibit strong self-splitting and it is clearly seen that the pulse evolves into \(n + 1\) subpulses upon propagation, although the system is completely linear.

It is worth noting that far-zone criteria of the type \(z \gg z_T\) for partially coherent fields have been examined in the context of spatially partially coherent fields \[51,52\], but not (to our knowledge) extensively in the temporal domain. A typical characteristic of far-zone propagation, well known in the spatial domain, is that the field propagates without a change in its functional form but only in scale. This characteristic is evident for the temporal LGCSM fields illustrated in Fig. 5 at distances \(z \gg z_T\). Such a behavior is natural in view of the space-time duality in optics \[53,54\], which has been examined also in the domain of optics of coherent ultrashort pulses \[55\]. The numerical results in Fig. 5 evidence the applicability of space-time duality and the associated far-zone propagation criteria to pulsed optical fields with partial coherence properties.
defined by Eq. (38), we find the propagated MCF to be of the form

$$\Gamma(t_1, t_2, z) = \frac{\omega_0 B_1}{2\sqrt{\pi} B} \exp \left[ -\frac{i\omega B_1}{2B} (t_1^2 - t_2^2) \right] (-1)^n 2^{-n} \left( \frac{2\pi B}{n!} \right) \int_0^\infty \cdots \int_0^\infty d\tau_1 \cdots d\tau_n \exp \left[ -i\omega \sum \tau_i \right] \exp \left[ -\frac{1}{2} \sum \frac{\tau_i^2}{T_i(z)} \right] \frac{1}{\sqrt{2\pi T_i(z)}} \right]^n. \tag{57}$$

The average intensity of the HGCSM pulses in second-order dispersive media is then given by

$$I(t, z) = \frac{\omega_0 T_0}{2\sqrt{\pi} B} \exp \left[ -\frac{i\omega B_1}{2B} (t_1^2 - t_2^2) \right] \left( 1 + \frac{z^2}{T_0 z} \right) \left( 1 + \frac{z^2}{T_0 z} \right)^{-1} \cdots \left( 1 + \frac{z^2}{T_0 z} \right)^{-1} \left[ 1 + \frac{z^2}{T_0 z} \right]^{1/2}. \tag{58}$$

where

$$T_c(z) = T_c \left[ (1 + \frac{z^2}{T_0^2})(1 + \frac{z^2}{T_0^2}) \right]^{1/2}. \tag{59}$$

With $n = 0$ this reduces to Eq. (56) for GSM fields.

B. Propagation of Hermite-Gaussian correlated Schell-model pulses

Carrying out the same procedures for the HGCSM pulses defined by Eq. (38), we find the propagated MCF to be of the form

$$\Gamma(t_1, t_2, z) = \frac{\omega_0 B_1}{2\sqrt{\pi} B} \exp \left[ -\frac{i\omega B_1}{2B} (t_1^2 - t_2^2) \right] (-1)^n 2^{-n} \left( \frac{2\pi B}{n!} \right) \int_0^\infty \cdots \int_0^\infty d\tau_1 \cdots d\tau_n \exp \left[ -i\omega \sum \tau_i \right] \exp \left[ -\frac{1}{2} \sum \frac{\tau_i^2}{T_i(z)} \right] \frac{1}{\sqrt{2\pi T_i(z)}} \right]^n. \tag{57}$$

The average intensity of the HGCSM pulses in second-order dispersive media is then given by

$$I(t, z) = \frac{\omega_0 T_0}{2\sqrt{\pi} B} \exp \left[ -\frac{i\omega B_1}{2B} (t_1^2 - t_2^2) \right] \left( 1 + \frac{z^2}{T_0 z} \right) \left( 1 + \frac{z^2}{T_0 z} \right)^{-1} \cdots \left( 1 + \frac{z^2}{T_0 z} \right)^{-1} \left[ 1 + \frac{z^2}{T_0 z} \right]^{1/2}. \tag{58}$$

where

$$T_c(z) = T_c \left[ (1 + \frac{z^2}{T_0^2})(1 + \frac{z^2}{T_0^2}) \right]^{1/2}. \tag{59}$$

With $n = 0$ this reduces to Eq. (56) for GSM fields.

Similar to Fig. 5, we show in Fig. 6 how the intensity $I(t, z)$ of HGCSM pulses with different orders $n$ evolves in dispersive media. We have used the same simulation parameters as before, and in this case the self-splitting mechanism is slightly different: with all $n > 0$ the pulse splits into two lobes, the separation of which increases with order $n$.

C. Interpretation

For both LGCSM and HGCSM pulses, the qualitative similarity of the mean intensity at a propagation distance $z = 20$ km and the power spectra is quite striking: the number of lobes in both are the same. It is indeed well known for coherent pulses that propagation in a dispersive medium into the temporal far zone transforms the temporal intensity profile into a functional form that is identical with the power spectrum of the pulses. The same is true for temporal 2F systems with matrix elements $A = D = 0$, $B = F$, and $C = -1/F$. A similar result applies to partially coherent pulse trains. To see this, let us evaluate the temporal intensity using Eq. (45) with $t_1 = t_2 = t$

$$I(t, z) = \frac{\omega_0}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Gamma_0(t_1, t_2) \times \exp \left[ -i\omega_0 \left[ A(t_1^2 - t_2^2) - 2\tau(t_1 - t_2) \right] \right] \, dt_1 \, dt_2. \tag{60}$$

In the case of a 2F system the first exponential term inside the integral is identically zero and in the temporal far field (Fraunhofer diffraction zone) it becomes insignificant. Hence, in these conditions,

$$I(t, z) = \frac{\omega_0}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Gamma_0(t_1, t_2) \times \exp \left[ -i\omega_0 \left[ (t_1 - t_2) \right] \right] \, dt_1 \, dt_2. \tag{61}$$

On the other hand, if we apply the Wiener-Khintchine theorem of Eq. (8) at $z = 0$, and set $\omega_1 = \omega_2 = \omega$, we find that spectral density is given by

$$S(\omega) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Gamma_0(t_1, t_2) \exp \left[ -i\omega(t_1 - t_2) \right] \, dt_1 \, dt_2. \tag{62}$$

Comparing these expressions we see that spectral density and the far-field (or Fourier-plane) temporal intensity share the same functional form, no matter what the degree of coherence of the pulses is.

We can see this equivalence explicitly for LSGM and HGCSM fields by considering the temporal far-zone limit $z \gg z_T$, which gives $T(z) \rightarrow T_0 z/z_T$, $T_c(z) \rightarrow T_0 z/z_T$, and $c(z) \rightarrow 1/2 T_0^2 z^2$. One can then see that, for HGCSM fields, Eq. (58) becomes functionally similar to Eq. (44). Correspondingly, for LGCSM fields, Eq. (52) can be shown to become functionally similar to Eq. (36).

V. CONCLUSIONS AND FINAL REMARKS

In the present paper, we have introduced LGCSM and HGCSM pulse models, which both exhibit temporal self-splitting upon propagation whenever the order index $n > 1$. 053838-7
This is explained by the distribution of the spectral density distribution, which has \( n + 1 \) lobes for LGCSM pulses and two lobes for HGCSM pulses. The self-splitting phenomenon was found to become particularly pronounced when the degree of coherence is reduced, but no self-splitting occurs in the fully coherent limit. The present study may have implications in optical data transmission since it opens up the possibility of encoding data into the temporal coherence function of the pulses. The information content encrypted in the coherence function would then unravel upon propagation in dispersive media.

Conversely, it is conceivable to construct initial pulse trains with a single-lobe (such as Gaussian) temporal coherence function, but with a multiply peaked intensity profile, in such a way that the power spectrum has only a single lobe. Such pulse trains would self-confine upon propagation in dispersive media.

Let us conclude with some remarks on experimental demonstration of the types of model fields considered here. One might start with a stationary, spatially coherent light source (a superluminescent diode, for instance) and use electro-optic modulation to produce a train of Gaussian Schell-model pulses with a low degree of spectral and temporal coherence [33]. Then the spectral modulation technique proposed in Ref. [43] could be applied to realize the initial pulse train with an appropriate temporal coherence function. In the cases considered here, this would require spectral amplitude modulation with binary phase modulation to access the negative values of the function \( g(\Delta t) \). Such modulation is, however, completely feasible. The modulated (spatially coherent) pulse train would be coupled into a single-mode fiber of suitable length to see the self-splitting effect.

Since the dispersion of most common single-mode optical fibers is around 40 ps\(^2\)/km, one would need several kilometers of fiber to be able to see the self-splitting effect. If one uses a telecommunication fiber outside of its specified operation wavelength, the dispersion increases so that the self-splitting would become apparent at much smaller fiber lengths. In this scheme a normal (fast) photodiode would be sufficient as a detector.

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**APPENDIX**

In this appendix, we list (for completeness) in order of their appearance some of the formulas used to derive the results in the main text [56]:

\[
L_n(x) = \frac{\exp(x)}{n!} \frac{d^n}{dx^n} \exp(-x)x^n, \quad (A1)
\]

\[
H_n(x) = (-1)^n \exp(x^2) \frac{d^n}{dx^n} \exp(-x^2), \quad (A2)
\]

\[
\int_{-\infty}^{\infty} H_n^2(x) \exp[-(x-y)^2] dx = 2^n n! \sqrt{\pi} L_n(-2y^2), \quad (A3)
\]

\[
\int_{-\infty}^{\infty} H_n^2(\alpha x) \exp[-(x-y)^2] dx = \sqrt{\pi} \sum_{k=0}^{n} 2^k k! \binom{n}{k}^2 (1 - \alpha^2)^{n-k} H_{2(n-k)}(\frac{\alpha y}{\sqrt{1 - \alpha^2}}), \quad (A4)
\]

\[
H_n^2(x) = \sum_{k=0}^{n} 2^k k! \binom{n}{k}^2 H_{2(n-k)}(x), \quad (A5)
\]

\[
\int_{-\infty}^{\infty} \exp[-(x-y)^2] H_{2n}(x) dx = \sqrt{\pi}(2y)^n, \quad (A6)
\]

\[
\int_{-\infty}^{\infty} x^n \exp[-(x-y)^2] dx = (2i)^n \sqrt{\pi} H_n(iy), \quad (A7)
\]

\[
L_n(x) = \sum_{q=0}^{n} \binom{n}{q} (-1)^q q! x^q. \quad (A8)
\]


