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# Three Main Paradigms of Simultaneous Localization and Mapping (SLAM) Problem

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## ABSTRACT

Simultaneous Localization and Mapping (SLAM) is one of the most challenging research areas within computer and machine vision for automated scene commentary and explanation. The SLAM technique has been a developing research area in the robotics context during recent years. By utilizing the SLAM method robot can estimate the different positions of the robot at the distinct points of time which can indicate the trajectory of robot as well as generate a map of the environment. SLAM has unique traits which are estimating the location of robot and building a map in the various types of environment. SLAM is effective in different types of environment such as indoor, outdoor district, Air, Underwater, Underground and Space. Several approaches have been investigated to use SLAM technique in distinct environments. The purpose of this paper is to provide an accurate perceptive review of case history of SLAM relied on laser/ultrasonic sensors and camera as perception input data. In addition, we mainly focus on three paradigms of SLAM problem with all its pros and cons. In the future, use intelligent methods and some new idea will be used on visual SLAM to estimate the motion intelligent underwater robot and building a feature map of marine environment.

**Keywords:** Simultaneous localization and mapping, localization, mapping, EKF, particle filter, graph-based SLAM.

## 1. INTRODUCTION

There is a growing notion that robots are assisting human in the different aspects. Robots are generally working in two categories of environment; constant and unstable. One of the most challenging problem in the robotics context is mapping the environment. When robots are working in the constant environment this problem is typically solved by just connecting the robot to the ground; therefore, they don't have localization and mapping problem because of not moving. This is the way that industrial robotics solve this problem such as robots for service tasks on factory floors. On the other hand, mapping the environment will become an issue when the robots move into the unstable environment; hence having a Simultaneous Localization and Mapping (SLAM) system [1] which solves mapping and localization at the same point in time. One of the advantages of this technique is that the procedure of mapping is online, therefore, the autonomous robot capable to detect all of the environmental factors which have innumerable intricate landmarks and obstacles [2]. Moreover, autonomous robots are able to explore in the new environment and make the right decision according to data from the new environment and navigate itself through the trajectory by developing the map from the new environment. The robot is able to estimate its own location and any obstacles, sans any information of new environment and human interference. By utilizing this map robots are able to navigate autonomously.

Nowadays SLAM algorithms are running into practical systems by applying modern methods of sequential Bayesian inference to estimate the state of landmarks in the environment. Autonomous robots are routinely achieved primary data from the landmarks and obstacles that are obtained by hardware apparatuses laser sensor and ultrasonic sensor. Furthermore, in recent years the utilization of cameras has been at the center of advancement in the autonomous robot. There are compelling reasons to consider the camera as an engaging option of SLAM sensor. One reason is that cameras are cheap and it has become universal. Another reasonable basis on 'intuitive appeal' as the sense human and animals primarily use to navigate [3]. Finally, it may simply be that accuracy of cameras enables the SLAM system to robustly operate in distinct environments. In the next step after interpreting raw data from the noisy sensor, constructing an explicit model of the world around (Mapping) as well as determining their own internal state (Localization) bring uncertainty problem. Uncertainty is one of the fundamental problems in autonomous robotics. "The two key computational solutions to the SLAM problem through the use of the extended Kalman filter (EKF-SLAM) and through the use of particle filters (FastSLAM)"[2].



shown in Figure 2, the robot observed landmarks very well in the beginning but after a while, when it drives on trajectory the robot estimation turned left owing to different environmental factor, but in the reality it has to turned right, therefore robot makes observation to find the landmark according to the map and correct its position by dragging towards its real position. The precision of position correction relies on accuracy of sensor and movement performance.

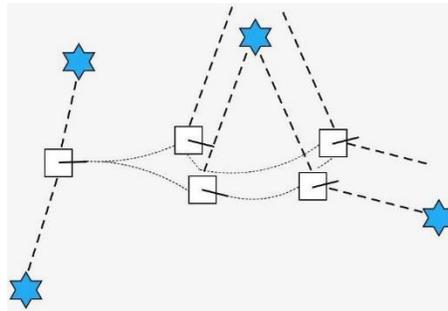


Figure 2. Localization Example.

### 2.3 SLAM

The main purpose of SLAM is to construct a map of environment and estimate the location of both, robot and the landmarks at the same time [10]. The interesting principle of SLAM is that the mapping estimation and localization accuracy rely on each other, which means a good map is essential to estimate the position of robot and landmarks, aside from that, the localization process is required to build a good map [11]. According to SLAM execution in Figure 3, the robot moves through the environment and estimates the location of landmarks and robot as well as building a map by utilizing commands and observation simultaneously. Robot uses sequence of control commands or odometry information [12], in order to enforce motions which are specified by  $u_{1:T}$ . Odometry information is a feedback from the system by using sensor information and it used because control commands suffer the lack of precision singly.

$$u_{1:T} = \{u_1, u_2, u_3, \dots, u_T\}. \quad (1)$$

In addition, robot requires observation data at different points in time which indicates the proximity to the closest obstacle or landmark that is identified with  $z_{1:T}$  [13]. Observation data may be obtained in an image from a video camera or from a laser scan regarding to robot sensor [14], [15].

$$z_{1:T} = \{z_1, z_2, z_3, \dots, z_T\}. \quad (2)$$

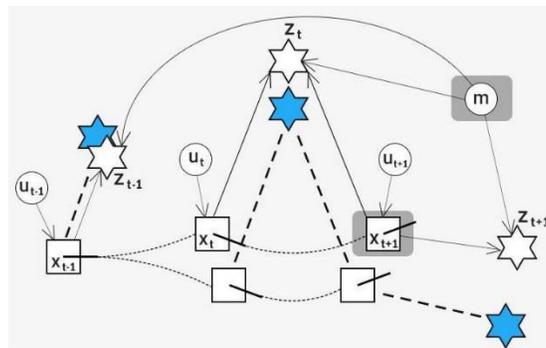


Figure 3. SLAM Example.

On the other hand, further steps are building a map of environment that is identified with  $m$ , and also estimating the path of robot since all the indispensable knowledge are received and defined. Path of the robot is defined as:

$$x_{0:T} = \{x_0, x_1, x_2, \dots, x_T\}. \quad (3)$$

As Figure 3 shows these data are not free of errors, like uncertainty in movement of robot and observations; therefore, use probability approaches to explicitly illustrate the model of uncertainty and take this model into account [16]. The probability distribution estimates the robot's trajectory and map of environment  $m$ , given the observations and control commands, probability distribution is defined as:

$$p(x_{0:T}, m | z_{1:T}, u_{1:T}). \quad (4)$$

The probability distribution in Full SLAM [17], estimates the entire trajectory of the robot. Although, sometimes robot seeks to regain and build a map up to the most recent pose which is called Online SLAM [18], [19] and defined as:

$$p(x_t, m | z_{1:t}, u_{1:t}). \quad (5)$$

Online SLAM is shown in Figure 3, it is used when the robot wants to make decisions based on the current position.

Taking into consideration, all of SLAM approaches, it exploits two significant concepts. The first concept is called the motion model which represents the distribution of relative motion of the robot given the old position of robot and control data.

$$p(x_t | x_{t-1}, u_t). \quad (6)$$

The second concept is called the observation model, which indicates distribution about the observation measurements given the robot's position and map.

$$p(z_t | x_t, m). \quad (7)$$

Motion and observation model can be divided into different types of representation like Gaussian model and Non-Gaussian model. In addition, Standard Odometry model represents motion between two positions in the plane by two rotations and a translation [12].

### 3. PRINCIPAL PARADIGMS TO SOLVE SLAM PROBLEM

One principal challenge with SLAM is the uncertainty [20], [21] in measurement which occurs due to exhaustively bad data association or observation noise that received from the sensors [22], [23]; therefore, it needs to apply probabilistic techniques to model this uncertainty suitably and take this uncertainty into account. Accordingly, three distinct paradigms are used in tackling this problem [24], first is the Extended Kalman Filter [25] which is made for Gaussian distributions, and second is Particle Filter based techniques which can have multimodal distributions, and finally we have Graph-based filter algorithm which creates a graph to address the SLAM problem.

#### 3.1 Extended Kalman Filter

Simultaneous Localization and Mapping explicates position estimation of robot and map of the environment which is known as state estimation problem [26], [27]. Bayes technique is one method to do state estimation given observations and control data [28], [29]. In an effort to solve the probability distribution (5), using a recursive equation that allows the system to incorporate an observation and one control at a time and estimate the current state of system recursively [30]. In addition, Extended Kalman Filter (EKF) is a recursive Bayes filter and this recursive equation can be divided into two steps [31], [32]. First, prediction step that estimates a new position of robot by taking into account the control command which was executed as well as current position of robot (8), and second step is called correction which considers current sensor observation along with predicted observation then based on the discrepancy of them and normalization constant  $\eta$ , correct the new state (9).

$$\overline{bel}(x_t) = \int p(x_t | u_t, x_{t-1}) \times bel(x_{t-1}) dx_{t-1}. \quad (8)$$

$$bel(x_t) = \eta \times p(z_t | u_t, x_t) \times \overline{bel}(x_t). \quad (9)$$

The EKF SLAM technique is effective solution to solve most realistic challenges in robotic nonlinear functions. The EKF SLAM algorithm relies on two steps which are prediction and correction steps [33]. In the prediction step, it estimates the predicted belief about the state by calculating the mean and the covariance matrix of state. The mean estimate uses

nonlinear function 'g', to estimate the new state by applying control commands and previous estimate about the state. The mean estimate equation defined as:

$$\bar{\mu}_t = g(u_t, \mu_{t-1}). \quad (10)$$

The covariance matrix is shown in equation (11) estimates uncertainty by multiplying the Jacobian of the motion model by previous uncertainty estimate and adding the noise  $R_t$  that is occurred to the process through the motion command.

$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t. \quad (11)$$

The second part of EKF SLAM algorithm is correction step which involves Kalman gain, correction of the mean estimate and correction of covariance matrix. Kalman gain calculation designates the uncertainty associated with an observation pursuant to the uncertainty of sensor observation  $Q_t$ , and Jacobian of the observation model  $H_t$ . Hence, when the uncertainty of sensor observation is large then the Kalman gain would be very small.

$$K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}. \quad (12)$$

$$\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t)). \quad (13)$$

$$\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t. \quad (14)$$

The correction of mean estimate in equation (13) is calculated by multiplying Kalman gain by discrepancy between obtained observation  $z_t$ , and the predicted observation  $h(\bar{\mu}_t)$ , and surcharging to predicted mean. Eventually, equation (14) calculates correction of covariance matrix taking into account the Kalman gain as well as the Jacobian of the sensor model and the covariance matrix of predicted belief. As the robot moves through the environment, prediction and correction steps will be accomplished consecutively to estimate an accurate map of the environment. There are various algorithms that can be actuated in the identical purpose and each of them will represent their own level of precision such as Unscented Kalman Filter (UKF) [34], Extended Information Filter (EIF) [35] and Compressed Extended Kalman Filter (CEKF) [36].

### 3.2 Particle Filter

Particle Filter algorithm [37] has been derived in order to solve state estimation when there is arbitrary distribution as well as robust nonlinear motion [38]. In addition, Particle Filter is a nonparametric variant of recursive Bayes filter [39], [40] that uses random samples to describe arbitrary distributions. In other word, Particle Filter utilizes set of samples  $X_t$ , to represent the posterior and every sample consists of two ingredients which are state hypothesis  $x^{[j]}$ , and importance weight  $w^{[j]}$  (15). The probability distribution is given by sum of all weighted samples (16), where  $\delta_{x^{[j]}}(x)$  denotes the Dirac delta distribution located in the state of sample.

$$X_t = \left\{ \langle x^{[j]}, w^{[j]} \rangle \right\}_{j=1,2,\dots,N}. \quad (15)$$

$$p(x) = \sum_{j=1}^N w^{[j]} \delta_{x^{[j]}}(x). \quad (16)$$

Particle Filter consists of three steps which are sampling step from proposal distribution, importance weighting and resampling. The algorithm that the Particle Filter use to drive theses three steps is delineated briefly as follows [41]: In the first step, importance sampling technique applies to generate samples for arbitrary distributions. This technique generates samples from target distribution  $f$ , by using a distinct distribution which is called the proposal distribution  $\pi$ .

$$x_t^{[j]} \sim \pi(x_t). \quad (17)$$

The second part of the algorithm is the correction step which calculates the importance weights by considering the differences between proposal distribution and target distribution at the sample location.

$$w_t^{[j]} = \frac{f(x_t^{[j]})}{\pi(x_t^{[j]})}. \quad (18)$$

The final step of Particle Filter algorithm is resampling, as done in GMapping system [42]; this redistributes samples by picking sample  $i$ , and replacing it into the resulting sample set  $X_t$ , the probability of picking a sample is proportional to its importance weight  $w_t^{[j]}$ . Particle Filter technique is efficient for low dimensional spaces, in contradistinction to high dimensional space owing to covering the probabilistic regions of the state space with samples. Accordingly, there are some other algorithms that can be used for the same purpose such as Fast-SLAM [43], [44] and Grid-based SLAM [45]. Fast-SLAM integrates a particle filter to estimate the robot's position and Extended Kalman filter for every pair of landmarks and particles. Fast-SLAM developed to the second version of Fast-SLAM which is called as Fast-SLAM 2.0 [38].

### 3.3 Graph-based SLAM

Graph-based technique [4] represents a simplified graph for an overdetermined system by minimizing the raw sensor measurements. The main concept of graph-based approach is to reveal the configuration of nodes that curtail the error represented by constraints. The graph contains particular nodes  $x_i$ , and every node in the graph corresponds to position of the robot at time  $t_i$ , during the mapping process [1], [3]. The raw measurement  $z_i$ , between two positions of robot is demonstrated by the edge in the graph which corresponds to a spatial constraint between two nodes. These constraints which originate from the measurement between individual nodes, are inherently uncertain based on relative transformations between two positions of robot. These transformations mainly rely on two types of information which are received from odometry measurement or are defined by aligning the observations obtained at the two different locations of robot. In the first type, the edge is generated from the odometry measurement between sequential robot positions that robot uses to move from  $x_i$  to  $x_{i+1}$ . In the second type, when the robot observes the same part of the environment, an edge relates the current position of robot  $x_i$  with the previous position that robot has been  $x_j$ , and based on the alignment of these observations constructs a virtual measurement between the position  $x_i$  and  $x_j$ . Homogeneous coordinates [5] are derived, in order to compute the transformation based on odometry edge (19), and observation edge (20). Evidently, these transformations can be contradictory since observations are always affected by noise; therefore, use information matrix  $\Omega_{ij}$ , to explicitly illustrate the model of uncertainty and take this model into account.

$$\left( X_i^{-1} X_{i+1} \right). \quad (19)$$

$$\left( X_i^{-1} X_j \right). \quad (20)$$

Figure 4 indicates a portion of graph representation of a SLAM process. The edge  $\bar{z}_{ij}$ , represents spatial constraint given a configuration of the positions  $x_i$  and  $x_j$ . This edge originates from mean measurement  $\bar{z}_{ij}$ , and the information matrix  $\Omega_{ij}$ , of virtual measurement. The error function  $e_{ij}(x_i, x_j)$ , computes the discrepancy between the predicted observation  $\bar{z}_{ij}$ , and the real observation  $z_{ij}$ . The purpose of Graph-based approach is to find the configuration of the robot poses  $\bar{x}$ , which minimize raw sensor measurements and obtain the best constraints, it can be defined as:

$$\bar{x} = \arg \min_x \sum_{ij} e_{ij}^T \Omega_{ij} e_{ij}. \quad (21)$$

Thus, in graph-based SLAM, construction the graph from the constraints is typically called front-end [7], [8] and it relies on sensor data, while determining the configuration of the robot's positions is called back-end [6] and depends on a short representation of the data which is sensor agnostic [2].

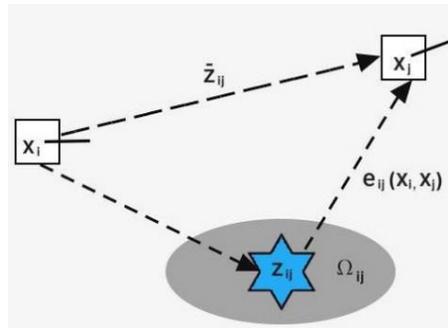


Figure 4. A portion of pose-graph representation of a SLAM process.

#### 4. CONCLUSION

The development of novel robotic applications has revealed the necessity of autonomous robots. In an effort to construct autonomous mobile robots, it has to solve the navigation problem. Various studies have been constructed by researchers to reveal the best approach in realizing an intelligent mobile robot. Along the way, there are some further observations to be made that require finding the best technique in realizing an intelligent mobile robot. While the SLAM community has seen great progress over the last few decades. Simultaneous Localization and Mapping technique ascertains that it is possible for the intelligent mobile robot to execute the mapping and localization process at the same time. This technique increases the robot's efficiency of task performance while exploring a dynamic environment. In this technique, accuracy localization is often accomplished by adjusting current sensor data to a high explanation map of the environment that is constructed in advance.

In this paper, we provide an intuitive review on the recursive Bayesian equation of the SLAM problem which estimates locations of landmarks and positions of robot are obtained by applying probability distributions. In the next part, we continued to survey three main paradigms of the essential SLAM algorithm in Extended Kalman Filter, Particle Filter and Graph-based SLAM which are three of the prominent solutions to address the SLAM problem in different environments. First, we look into the Extended Kalman Filter algorithm which is kind of the first prosperous system that it is used in SLAM to estimate the location of the robot as well as a map of the environment; then, we look into Particle Filter approach which is used for localizing robot in an environment given a created map; finally, last but not least the Graph-based SLAM technique which is separated in two steps: creating the graph from the raw measurements and determining the configuration of the robot's positions given the edges of the graph.

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