2017

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Informa UK Limited

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http://dx.doi.org/10.1080/00313831.2017.1306797

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Students Acquiring Expertise through Student-Centered Learning in Mathematics lessons

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Abstract
This article reports the results of a study that investigated junior high school students’ experiences with learning in mathematics lessons that were based on self-guidance, use of technology, and minimalist instruction. The study was part of a ClassPad project and data were obtained from reports written by the 23 students after the ClassPad project ended. A model describing the student’s process of mastering doing and learning mathematics through acquiring expertise processes was constructed by using grounded theory method. The mastery of doing and/or learning was reached either with satisfaction or dissatisfaction. Two different learning profiles, one concluding with students feeling satisfied with their learning and the other concluding with students feeling unsatisfied illustrated the students’ typical processes. The findings further revealed that when the teacher’s role was minimized and the students had the opportunity to self-guide their learning in an environment with various materials and easy-to-use technology, student-centered learning occurred.

Keywords Learning mathematics · Student-centered learning · Technology · Minimalist instruction
Introduction

Over the decades, the general paradigm of teaching and learning has been changing in mathematics, as in many other fields of education. There has been a shift from the behaviorist knowledge transmission model to the constructivist paradigm, which emphasizes cognitive processes. In addition, the socio-constructivist (socio-cultural) paradigm, which emphasizes social factors in constructing shared knowledge has had implications for teaching and learning (Ravitz, Becker and Wong 2000; Lerman 2001; cf. Brown 2001). In the research on teaching practices in mathematics, one of the focuses has been on collaborative problem-based methods for improving the learning process and outcomes and students’ control over learning (Fransisco 2013; Lampert and Cobb 2003; McCrone 2005; Sfard 2001). For example, Qin, Johnson, and Johnson (1995) showed that collaborative problem-based learning improves learners’ abilities to solve, administer, and memorize mathematical concepts and principles. In addition, this type of learning provides more resourcefulness and advances the power of reasoning (cf. Derting and Ebert-May 2010). However, the traditional teaching approach is still thriving (World Economic Forum 2015), especially in the teaching of mathematics (cf. Silfverberg and Haapasalo 2011; Hiebert et al. 2003; Hollingsworth, Logan and McCrae 2003); thus, the need for a cultural shift toward student-centered approaches is still a topical issue (Alliance of Excellent Education (AEE) 2012; Finnish National Board of Education 2016). The purpose of this article is to describe the results of a research project that investigated junior high school students’ experiences with learning in mathematics lessons that were based on self-guidance, use of technology, and minimalist instruction (MI), a teaching method that aims at enabling student-centered learning (SCL). In this case, the students were asked to learn the linear function.

SCL is a complicated concept that has been interpreted in several different ways. According to O’Neill and McMahon (2005), the interpretation of this term varies between authors, as some equate it with “active learning,” whereas others take a more comprehensive definition, including active learning, choice in learning, and the shift of power in the teacher-student relationship. Some literature on SCL highlights students’ autonomy (Burnard 1999; Zimmermann 2003; Gibbs 1995; Wu and Huang 2007) whereas many others argue that SCL’s tenets establish it (Lea, Stephenson and Troy 2003; Brandes and Ginnis, 1986) as opposite to the teacher-centered approach (Kember 1997; Harden and Crosby 2000). According to Neumann (2013), however, SCL contexts can be said to be centered in students, on students, or with students. Learning contexts that are centered in students imply that learning dwells within the student and is generated with little or no assistance from anyone else. Within this context, educators simply must not inhibit learning, for learning essentially becomes self-generating and self-propelling. The second relationship, in which teaching is “analogous to a kind of handicraft work,” is assigned the preposition “on.” Here, teachers determine students’ broad educational needs and students react to those plans and prerogatives. In this context, a teacher’s ability to convey necessary material and a student’s responsibility to learn
it are essential for learning. The third relationship, involving free beings, is assigned the preposition “with.” Learning contexts centered with students bring the teacher into partnership with the student, implying that learning emerges as teacher and student collaborate.

Overall, student-centered approaches offer students opportunities to collaborate and cooperate as well as to self-guide in making decisions regarding their own processes. These approaches are not permanent, but active participants work cooperatively and collaboratively at different times during the learning process (Holliman and Scanlon 2006). According to Roschelle and Teasley (1995), cooperative work is accomplished by dividing the labor among participants as an activity in which each person is responsible for a portion of the problem-solving, whereas collaboration involves the mutual engagement of participants in a coordinated effort to solve the problem together. Self-guidance (Turner 1995), means learners participate in guiding and planning their own learning, like defining homework. These elements improve the learner’s intrinsic motivation toward learning and can be seen as creating better learning results. Many have reported that the feeling of self-guidance improves learners’ meta-cognitive skills and ability to evaluate their own work (Immordino-Yang and Sylvan 2010; Kauffman 2001; Wright, Horn and Sanders 1997). Conversely, the teacher-centered, well-controlled approach decreases learners’ intrinsic motivation toward and involvement in working (Patall, Cooper and Wynn 2010). The term “self-guidance” refers to similar processes as the concept of self-regulated learning (cf. Boekaerts and Niemivirta 2000; Zimmermann 2002) promoting an opportunity to consider the process phases (preparatory, performance and appraisal) with different emphasis (Puustinen and Pulkinen 2001).

Several approaches supporting SCL have been developed, such as discovery learning (Anthony 1973; Bruner 1961), problem-based learning (PBL) (Barrows and Tamblyn 1980; Schmidt 1983), inquiry learning (Papert 1980; Rutherford 1964), experiential learning (Boud, Keogh and Walker 1985; Kolb and Fry 1975), and constructivist learning (Jonassen 1991; Steffe and Gale 1995). According to Kirsch, Sweller, and Clark (2006), these are minimally guided approaches. The minimally guided approach contains two main assumptions: 1) students solve “authentic” problems or acquire complex knowledge in information-rich settings and 2) students acquire knowledge through experience based on the procedures of the discipline. In addition, minimal guidance is offered in the form of process- or task-relevant information that is available if learners choose to use it (Kirschner et al. 2006).

An example of the minimally guided approaches is Minimal instruction (MI) that was created by Carroll (1990). MI is a teaching method that differs radically from traditional direct teaching; it fosters pure SCL. MI has the following specific principles: 1) specific content and outcomes cannot be pre-specified, although a core knowledge domain may be specified; 2) learning is modeled and coached for students with unscripted teacher responses; (3) learning goals are determined from authentic tasks stressing doing and exploring; (4) errors are not avoided but used for instruction; (5) learners construct multiple perspectives or solutions through discussion and collaboration; (6)
learning focuses on the process of knowledge construction and development of reflective awareness of that process; (7) the criterion for success is the transfer of learning and a change in students’ action potential; and (8) the assessment is ongoing and based on the learners’ needs (Haapasalo 2007; cf. Lambrecht 1999). MI was applied in the study introduced in this article.

Technology can offer another pedagogical solution for enriching and changing the traditional method of teaching, as it can provide considerable autonomy for learners to determine when, where, and how to learn, which can enhance learners’ self-direction in actions (Beers, Boshuizen, Kirschner and Gijselaers 2007; Resta and Laferrière 2007; Järvelä, Hurme and Järvenoja 2011). Mathematics technology in particular offers new tools for learning. For example, Tall et al. (2001) asserted that advanced mathematical thinking involves a combination of geometric and symbolic thinking. Technologies such as computer algebra system (CAS) calculators provide a suitable environment for studying a new mathematical concept in the problem-solving process by manipulating geometric or symbolic presentations of the concept and researching the influence of manipulating each presenting form (Kadijevich and Haapasalo 2001). Yet, even though the advantages and possibilities of technologies to support learning are nowadays acknowledged, it is hard to change the traditions of teaching and learning at school. For example, according Kontkanen et. al (2016) students do not have the confidence to radically change learning styles to take advantage of the affordances of the devices. In addition, previous research on new technologies entering schools has shown that in most cases educators replicate old pedagogies on new devices (cf. Laurillard 2012; Reeves et al. 2011).

One important topic in mathematics education research has been the linear function. In particular, they have investigated the different strategies for learning the linear function (Moschkovich 2004; Breidenbach, Dubinsky, Nichols and Hawks 1992; Even 1990; Moschkovich, Schoenfeld and Arcavi 1993; Schwarz and Yerushalmi 1992; Sfard 1992). Concerning learning that happened in computer environments, study results show that when students first study lines, they tend to view non-horizontal parallel lines as horizontal rather than vertical translations of each other (Chiu, Kessel, Moschkovich and Munoz-Nunez 2001; Goldenberg 1988; Moschkovich 1999; Schoenfeld, Smith and Arcavi 1993). By considering new problems, students can specify when a strategy applies and experiment with refinements to overcome particular limitations. Moreover, Chiu et al. (2001) suggested encouraging students to develop and refine strategies associated with this concept rather than ignoring or trying to eradicate them.

The ClassPad Project

This article is based on the results of the ClassPad project (Eronen & Haapasalo 2011), which attempted to develop a model for mathematics lessons in which learning was based on self-guided learning, use of technology, and the teacher’s action via the principles of MI. The ClassPad project was part of the ongoing longitudinal Model Construction for Didactic and Empirical
Problems of Mathematics Education (MODEM) 1 project that aims to develop mathematics education within a specific socio-cultural framework by questioning the traditional roles of teachers and students and by emphasizing SCL (cf. Haapasalo 2003; 2007). The six-week project was carried out in the fall of 2005. In needs to be noted that even though the study was conducted over ten years ago, the article is very relevant today. First, the technology used in the project is still up-to-date. The ClassPad calculator was released in 2005 but it is still widely used in Finland. In addition, ClassPad calculator has been allowed to be used in the Finnish national matriculation examination no earlier than in 2012. Second, the pedagogical approach used in the study, i.e. MI that fosters student-centered learning, is very topical in Finland as the new National Core Curricula (Wwwophfi, 2016) emphasizes a collaborative atmosphere, as well as the promotion of student autonomy in studying. Thus, the results of the study will be useful when schools will start working according to the new curricula in autumn 2016.

The researchers carried out the principles of MI via the ClassPad project as follows. During the study period, the students had full freedom to self-guide their learning process, that is, the students could decide how they would work during the math lessons and neither instruction on the topic was given nor homework, controlled lesson tasks, or time limitations were set. Instead, at the beginning of the project, the students were informed that the goal of the project was to investigate how to learn without the teacher’s instruction, relying solely on the help of classmates and/or technology. The students were also informed that they could collaborate with their classmates during the process, but they were also allowed to work independently with the help of the CAS calculator (Casio ClassPad 300).

The ClassPad project lasted for nine lessons, and each lesson was 45 minutes long. The content to be learned was the linear function. The participating students, who were in the ninth grade, were not familiar with the content. The content of the project was developmentally suitable for the students, as linear function, was one of the core topics in the Finnish mathematics curriculum for ninth grades. Students were asked to learn how ClassPad did a geometric presentation from a given symbolic form and vice versa by using the drag-and-drop function on the ClassPad calculator. They could concentrate on any geometric figure they wanted, but only the materials (problems) for linear function were offered. The purpose of the lesson was to familiarize the students with the calculator and how to use it to solve the project’s problems. These problems were offered in the form of a “problem buffet,” in which the students had the freedom to choose which kind of problems they wanted to handle at the moment. At the end of the first lesson, the teacher briefly introduced how to use the problem buffet, but did not give any detailed information about the problems the students were supposed to solve with the help of the calculator. After the first lesson, the teacher did not give the students any more traditional instruction, but instead only provided support and advice if the students asked for them. During the rest of the sessions of the project, the students worked independently, in pairs, or in groups and solved problems that they had chosen from the buffet.
The problems (53) were created based on the MODEM 1 project framework (see Haapasalo 2007) and dealt with four problem categories: concept orientation and definition (8), identification (13), production (20), and reinforcement (13). The problems were created to interact within and between three presentations of form (symbolic, graphic, and verbal; see Fig. 1).

**FIGURE 1 HERE**

The goals of the orientation and definition problems were to offer interpretations of the situation based on the pupils’ mental models and to offer students opportunities to make their own investigations. The aim of the identification (I) problems was to train students to identify concept attributes in verbal (V), symbolic (S), and graphic (G) forms (six problem types: IVV, IVG, IVS, IGG, ISS, and ISG). The aim of the production (P) problems was for the students to use a given presentation of the concept to produce another representation in a different form (nine problem types: PGV, PGS, PGG, PSG, PSV, PSS, PVS, PVV, and PVG). The aim of the reinforcement problems (application) was to train and use concept attributes and to develop procedural knowledge to use in problem-solving and applications (cf. Haapasalo 2003).

To follow up on the students’ learning about linear function, the students were tested three times by using MODEM 1 framework test patterns (Eronen & Haapasalo 2011): before the project (pre-test), at the end of the project (test), and five months after the project (re-test). The Cronbach’s alpha calculated from all test items showed a good reliability level in all three test (see table 1). After the working period, the students’ total scores in test were significantly higher (p < 0.000, sign test) than in pre-test. Moreover, the students’ scores in the re-test showed that the student’s mastery of the concepts they had learned during the ClassPad project was still at a high level as there were no significant difference (p<0.383) in the scores in test and re-test. The test results indicated that the students indeed learned the concept of linear function during the ClassPad project (see more, Eronen & Haapasalo 2011).

**TABLE 1 HERE**

However, the positive results of the cognitive skills test in the ClassPad project somewhat contradicted Kirschner et al.’s (2006) empirical studies over the past 50 years. Their studies show that minimally guided instruction is less effective and less efficient than other instructional approaches (cf. Mayer 2004). Thus, investigating whether students’ personal accounts and experiences can help to understand what actually happened during the learning processes, and what might have contributed to the positive results in the process in particular, was important, as the research design was not completely controlled. In addition, it was assumed that the new teaching method and technology contributed to the students’ learning success. Thus, it was important to discover whether the students’ accounts increase understanding of the overall impact of MI on students’ learning processes.
Method

The research task of the study was to investigate the students’ learning experiences during the Classpad project and to develop a model of their processes based on the inductively analyzed data from the final reports. In other words, the goal was to gain the emic understanding of the students’ learning processes during the math lessons and to create a theory/model that would make the students’ implicit processes explicit. Thus, the “bottom-up” data analysis methods of grounded theory (Glaser & Strauss, 1967; Glaser, 1978) were applied in the study. The application of the method was shaped by the ideas of the original grounded theory approach (Glaser & Strauss, 1967) that introduced a flexible approach to data analysis. More precisely, the grounded theory approach used can be described as being based on the ideas of the Glaserian school that stresses openness in analysis process and creativity in the emergence of a theory rather than the Straussian school of the grounded theory method emphasizes the deliberate development of theoretical insights by using explicit coding paradigms and procedures.

Participants

The participants in the study were students in a Finnish regular junior high school ninth-grade class (23 students, 15 years old) whose math instruction was based on the guidelines of the Finnish national curriculum. The students had studied together before the project for two years, and thus, they knew each other quite well. Eleven students were male, and 12 were female. The content of the lessons, the linear function, was new to the students. The students’ math teacher also participated in the math lessons. His role was to facilitate the students’ learning processes when needed. The letter B in the annotated students’ comments in the results section of the article refers to a male participant (boy) and G to a female participant (girl). The number after the letter indicates the number of a participant of the study.

Data collection and analysis

Two weeks after the Classpad project ended, the students wrote personal reports on their experiences of the project during two native language lessons. Each lesson lasted for 45 minutes and the students had altogether 90 minutes time to write their reports. The data for this article were selected from the part of the reports (n=23) in which the students described their learning experiences during the project. The written instruction for this part of the reports was as follows: Write about your own experiences and feelings that you had during the project. You can for example

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1 An emic perspective attempts to capture participants' own meanings of real-world events (Yin (2010)).
compare them to previous experiences you have had during math lessons. Evaluate your own learning and describe your opinion of the work during the project. Describe moments when you were satisfied (like “a-ha” feelings) and moments when you were frustrated. The time for writing was not limited, but all students finished the report during two lessons. The reports were written by using the Microsoft Word program. The length of the parts of the reports that were analyzed for this article varied from two lines up to three pages of text (Font; Times New Roman, Font size; 12, Line spacing; 1.5). The total amount of data was 36 pages of typed text.

After rich data from the students’ personal reports were obtained, three analytic steps were conducted. In the open coding phase, the data derived from the students’ final reports were examined in detail and coded for emerging concepts. During the open coding, 343 concepts were identified. In the theoretical coding phase, the number of concepts was reduced to 183 and grouped in tentative categories. Patterns of the students’ processes in the data were identified, and the emerging categories and their relationships were carefully compared to make sure that the categories covered most of the variation seen in the data. Posing questions to the data was a central analysis strategy (Glaser, 1978, 123) used during the comparison. The comparison across the categories led to the formation of five categories, three subcategories and two modes (see table 2).

TABLE 2 HERE

The model describing the students’ mastery of doing and learning math was constructed and finalized. The model was tested and verified by reading the data of produced by each student and by checking how it fit with the model. During the verifying process two students’ profiles were discovered; L-profile students and V-profile students.

Results

According to the results of this study, the students’ experiences of the learning processes in the Classpad project can be described by a model that contains four categories contributing to the fifth category: the mastery of doing and learning math. The model consisted of five main categories. Each category contained two modes, doing and learning, and three subcategories, tools, work style, and content. The core category was acquiring expertise, which had three subcategories: acquiring expertise in the tools, acquiring expertise in the work style, and acquiring expertise in the content. The core category was related to the learner’s conception of himself or herself as a student in math lessons, the learner’s self-guidance during math lessons and the minimalist instruction environment for math lessons. Each category also had three subcategories. The outcome of the process of acquiring expertise was mastering doing and learning math with or without a feeling of satisfaction.

Students’ conception of themselves as students during math lessons

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The students’ conceptions of themselves as learners during math lessons was the first element identified to contribute to the students’ acquiring expertise in mastering doing and learning math. The students’ conceptions were related to the tools, work style, and content of the math lessons. Many students wrote that they felt a lack of overall skills for using technology, and it slowed down the use of the ClassPad calculator in the beginning of the project. However, most of the students described having learned to use ClassPad during a couple of lessons: “In the beginning, I had difficulties learning to use the calculator, but overall, it was easy to learn to use” (P4). Only a few comments were found in the data in which the students questioned the use of ClassPad even as a tool for doing math, that is, facilitating the completion of a math task.

The students’ acquirement of expertise in work style was influenced by their conceptions of their work style in an ordinary math lesson. According to the data, the students identified themselves mainly as doers rather than learners during an ordinary math lesson: A teacher taught, and the students were expected to take notes in their notebooks and to work mainly alone. Interestingly, the work style of the Classpad project was experienced in a positive way, particularly by students who felt they were doers rather than learners in ordinary math lessons: “A good thing [in the project] that supported my learning was that I did not have to listen to the teacher’s teaching that I did not understand but that I could teach myself by using the speed and difficulty level that was good for me” (B3).

The students’ acquirement of expertise in content during the project was also connected to the students’ thoughts about their overall interests and abilities to acquire the content during math lessons. Many students indicated that their motivation to do and learn math during ordinary math lessons was quite low. “If we had worked by using traditional methods, I am sure that I would not been motivated to study so much and the level of learning would not have been the same” (B2). However, the work style and the technical tool used in the project increased the students’ interest not only in doing math but also in learning it.

“Working was interesting ... and for once I felt that I had learned something. It is important that when you are learning you do not work all the time the same way. In other words, variety is the spice of life. ClassPad helped me learn easily” (G12).

Many students also wrote that they became more aware of their abilities to do and learn math content than before and that it was easier or more fun to learn math in the project than normally in math lessons. However, two out of the 23 students felt that they had difficulty not only learning but also doing the project math tasks: “The fundamentals were easy to solve with a calculator, but the more difficult tasks were too much for my brain capacity. The task started to irritate me as I did not understand what I was supposed to do” (B4). Thus, the project did not strengthen the
students’ conception of themselves as doers or learners in math lessons. Instead, the learning process was problematic and frustrating.

Self-guidance in math lessons

According to the data, self-guidance in relation to the tool, work style, and content was a crucial element contributing to mastering doing and learning math. Self-guided use of the tool, i.e., calculator, was successful for every student. Opinions such as “In the beginning, the use of the device was really boring, and it caused me a lot of problems as I was not able to use it. It caused me a huge aha moment when I learned to use the device and to do exercises with it” (B11) showed that the ClassPad was easy to adopt independently, even if the students had concerns about their abilities to use technical tools in general. In addition to the tool, the students had to learn to undertake self-guided work during the project. The process included many phases as described in an example below.

The beginning [of the process] started well, after that speed [of working and learning] slowed down. Problems that we were able to solve were quickly solved but more difficult problems remained partly unsolved. On the 28th of 11 I learned to define the slope and thus, I was able to write down how the equation of straight line is defined. During that moment, the joy of learning was the greatest. Studying continued in a familiar but unstable manner and by the end of the learning period, we were able to solve almost all problems that were available in first lessons and even some problems that were available later during the project. (G2)

In the beginning, many students were unsure about the goals of the project, i.e., what to study. As the teacher did not teach, he was available only if help was needed. The students had difficulty setting goals for working and learning independently. Additionally, as the teacher did not give feedback, the students had to figure out by themselves or with their classmates what and how they had learned. Therefore, the students wrote, for example, that they had been able to do math exercises but had not been sure whether they had really learned math:

I learned issues well during the period, at least in my mind; nevertheless, I never felt quite sure about it. When I investigated all issues by myself or with my friends, some issues remained unclear even though I was able to apply them when doing exercises. (G10)

Regardless of the uncertainty of their learning, the students described that overall self-guidance during math lessons was a positive experience and that it could be acquired. In particular, self-guided management of the timetable evoked strong positive feelings:
The independent work style appeared nicer to me than working with classmates as I could do exercises at my own pace. I do not have patience to work with exercises for a long time, but I like to move on as soon as I have learned the issues. (B6)

The expectation of the students’ self-guidance during math lessons also had an impact on the way the students worked with the problems they had selected from the buffet. The intensity of the self-guided problem-solving process varied among the students and in different lessons. During some lessons, the students dealt with many problems. However, when the students did not solve some problems, it caused frustration and decreased their interest in continuing to work with other problems. However, students described that they were able to overcome their frustrations and lack of motivation to continue by seeking help from Classpad calculator or by getting support from their peers.

If I did not understand something after hard thinking, my motivation quickly declined and I kind of gave up and stopped trying and working. I found Classpad [calculator] and my peer’s [G2] help very useful. The nicest thing was to understand and discover a new issue that had troubled me a lot. Issues related to equation and slope did not interested me at all. [However] when I for example understood many issues on slope during one session I got exhilarated. (B2)

The self-guided selection of problems from the buffet varied among the students. Some students mentioned that they just randomly picked problems from the buffet, and some students carefully selected what kinds of problems they were able to solve: “I understood that it was not yet time to solve the applied problems, and I moved back to do the orienting tasks” (G1). Overall, the students’ skills in guiding their own doing and learning processes developed during the project: “I may have learned more efficiently as I needed to take more responsibility for my learning” (G6). Even though the students were worried at the beginning of the project about not having a teacher control their learning processes, by the end, the majority of the students (n=21) realized that self-guidance facilitated rather than inhibited their doing and learning math.

**Minimalist instruction environment for math lessons**

The minimalist instruction environment was framed to the students’ learning during the Classpad project. When the students wrote about the tool in relation to minimalist instruction, they described that doing math with ClassPad provided a nice change compared to the ordinary means of learning math and the calculator was a very usable tool for learning math compared to a notebook:

The calculator was a useful tool particularly in a situation if something went wrong; one just pressed the delete button, and the line disappeared. Had we used
notebooks, we would have needed to erase far too much. Thus, the calculator cut down extra work. (B8)

The way the students were expected to work during the lessons in the project and how the teacher was involved in the process were new to the students; the teacher did not teach them actively. Thus, it took some time and effort for the students to learn to do and to learn math without a teacher’s instruction: “In the beginning, I sank my teeth into work lightly, but when I realized that this way of working requires insight, I started to work harder” (G2). After some time, the minimalist instruction did not feel as special anymore, and some students even did extra work at home as they wanted to do things well.

As the teacher did not teach during the lessons, the students’ style of teamwork was either cooperative or collaborative. The students selected cooperation if they noticed that it was not necessary to get help in order to do or learn math but rather to share tasks and get the problems solved. However, as work during cooperation was not always mutually agreed on sometimes the students worked also at their own pace and asked for and gave help to teammates or other students only when necessary. Collaboration, on the other hand, meant that the students worked closely together and felt they were equal team members. Teacher did not guide the process but the team members decided jointly how to collaborate. According to the results, collaboration appeared to be particularly significant in supporting the students’ mastery of doing and learning math. The students felt that one of the most rewarding aspects of teamwork was teaching and learning from each other and in particular completing a difficult task together: “Working together felt good. Working with a friend allows one to teach for example how to use a graphing calculator or how to solve a problem if the other person does not master them” (B2).

In addition, the students who mainly collaborated during the process also engaged in some cooperation. Over time, almost all of the students worked to some extent as teams, and even six out of the seven students who at the beginning of the project decided to work alone used the benefits of teamwork. For example, a student who in the beginning wanted to work alone wrote at the end of the project: “Pair and teamwork were useful ways of working for me in terms of learning as my classmates could help me a lot to understand issues” (G5).

Minimalist instruction also had an impact on the process of acquiring the content of the lessons. The students made a distinction between doing and learning problem solving. If the students solved the problem, but did not thoroughly understand what they did, they did problem solving. However, the students described that after several lessons they usually solved the problem and eventually understood and learned problem solving, which made them very satisfied, happy, and highly motivated to continue the learning process: “The definition of the slope caused maybe us problems and headache the most. The moment when we were able to solve it was very joyous! Success gave us new enthusiasm to learn” (G2).
Two students reported that they did but did not learn math during the project (B4 and G7). To them, a project following the principles of minimalist instruction did not give the type of support they were accustomed to. Thus, doing and learning math self-guided without a teacher’s instruction was not satisfying for them and did not facilitate their learning processes.

**Acquirement of expertise**

Acquiring expertise had three subcategories, acquiring expertise in the tool, work style, and content. Expertise in the tools was acquired when the students were able to use the device and solve the math problem with the help of ClassPad. Acquiring expertise in the tool was the easiest field of expertise mastered by all students, even though it was frustrating and slowed down their process in the beginning: “Working appeared at the beginning frustrating because I was not able to use or make the calculator to function the way I wanted. It took many hours merely to learn to use the calculator” (B7). Nevertheless, as acquiring expertise in the tools was easy, it gave them a feeling of progress and satisfaction compared to learning with traditional tools: “ClassPad was an interesting tool and promoted learning, and it was more fun to work with it compared to traditional working methods” (G12). However, two students (B4 and G7) did not find ClassPad a suitable and satisfactory device to learn math, even though they used ClassPad successfully during the project.

The subcategory acquiring expertise in work style included students’ general comments on the study process during the project. Acquiring expertise in the work style appeared in the data as an individual step-by-step process that lasted until the end of the project. The two central elements in acquiring expertise in the work style were self-guidance and teamwork. Acquiring expertise in the work style required self-guidance, which the students found difficult to learn at the beginning of the project:

> It was difficult and unclear to start to work in the beginning as I did not know what I was supposed to do and where to start from. But when I noticed nobody knew anything more than I did I started to work on exercises. (G1)

Over time, the students learned to work self-guided concerning the aims and schedule, and thus, they acquired expertise in the work style. In addition, they evaluated the suitability of the work style for themselves. Most of the students liked the new work style. However, according to one student, a self-guided work style did not push her to work as much as the traditional work style. Thus, she preferred the old work style, i.e. a teacher teaching and students learning: “I noticed that I worked less than before as nobody was pushing me and I did not do any homework at home” (G11). On the other hand, self-guided work style also pushed some students to work harder than usually.
I worked a bit less than G2, but more than usually during math class. I was irritated that I could not doze on my desk even though I was tired. It would have kept me behind G2 and our project would not have succeeded. (B2)

Another central element for acquiring expertise in work style was teamwork. Students who decided to work in teams from the very beginning of the project acquired expertise in the new work style successfully within quite a short time. They chose the problems to solve together and worked them through in collaboration with team members. Students working in teams were also satisfied with their learning processes. However, there were many obstacles during the process of acquiring the expertise of the new work style. When the students had full autonomy to guide teamwork, they had to negotiate for shared understanding of the goals of their work. Occasional disagreements among the team members caused dissatisfaction. However, through those conflicts, the students learned to work collaboratively and acquired expertise in the work style.

For students who decided to work alone, the process of acquiring expertise in the work style was different. At first, they tried to solve problems by themselves with the help of ClassPad. However, when difficulties arose and there was nobody to help, the students started cooperating with other students. As they had selected problems suitable for them, not for the team, their work with the team occurred occasionally and was based on individual needs for support. Thus, the students’ teamwork was cooperative rather than collaborative, but was nevertheless important for their learning: “If we had disagreements, we started to discuss whose interpretation was right, and the issue impressed itself on my mind and was not forgotten even the next day” (G1).

Overall, the students’ expertise in the new work style increased during the project. “It was interesting to see how I learned by using a more independent working method” (G6). In addition, some students also wrote that the new work style had been an extremely positive experience that contributed to acquiring expertise in the content: “For once I felt that I had learned something” (G12). However, despite the many positive comments on the work style, a few students were disappointed in not being able to acquire expertise in the work style and thus being unable to solve math problems.

Acquiring expertise in content was the third subcategory of the core category, acquiring expertise. Expertise in content was acquired when the students were able to solve and understand the solving process for the problems. Some students were satisfied with acquiring expertise to do content, i.e., to be able to solve problems selected from the buffet. If the problem was too complicated to solve, the students got frustrated. However, problems that took many lessons to solve successfully gave a lot of self-confidence:

It was particularly nice when, for example, exercises that had caused difficulties and frustration in the beginning and thus had been left unsolved were solved at
the end of the project. And at that moment, one really realized that there has been progress in studying. (G1)

For students who aimed at acquiring expertise in learning math content, expertise in doing problem solving was not a satisfying goal, even though they were able to acquire it. The students acquired expertise in learning math content when they really understood the solution to a problem, for example, the meaning of a new concept, slope:

After the first small a-ha moment, my motivation started to increase. I realized that calculation had been useful. Quite soon after that, we learned a lot of new things about other concepts related to the slope equation, and it increased the meaningfulness of the work considerably. It was nicer to do the exercises afterwards. I knew what they were all about and I was able to validate issues. (G8)

Such experiences positively influenced students’ conception of themselves as students in math lessons.

**Modes of mastery and satisfaction**

The processes for acquiring expertise led to four modes of mastery as an outcome: 1) mastering doing math but not mastering learning math, 2) mastering doing and learning math, 3) mastering neither doing math nor learning math, and 4) not mastering doing math but mastering learning math. All four modes of mastery included the satisfaction–dissatisfaction dimension, that is, how satisfied or dissatisfied students felt with their mastery outcomes and subcategories tool, work style, and content. Furthermore the students’ shifts from one mode to another during the project were identified from the data.

At the beginning of the project, all the students started from mastering doing but not mastering learning the math mode, and felt satisfied mainly because they were excited about the new work style and mastering the new tool; i.e., ClassPad was easy to adopt. After some lessons, all the students were still at the level of mastering doing math, but their satisfaction decreased as mastering the new work style was an ongoing process and they did not master learning the math (1). Toward the end of the project, however, most of the students reached the mode of mastering doing and learning math, and they were satisfied if the learning was based on either mutual collaboration or autonomous work in content (2). In contrast, even if the students had mastered doing and learning math, they were unsatisfied if their learning process had been guided by unilateral cooperation with peers or the teacher (3).
Mastering learning without mastering doing math was possible if the student had already mastered everything needed in the beginning of the project. The mode was satisfactory in the beginning of the process, as there was no need to do anything. However, over time, the students’ satisfaction decreased as there was nothing to do or learn. However, this mode was not very typical and was identified from the data adopted from only one student’s report.

Mastering neither doing nor learning math appeared in situations when the students stopped working. The situation became possible in the project when the students had the autonomy to guide their own doing and learning. The students’ satisfaction in this work mode increased if the reason for entering this mode was due to personal or non-content-related issues that they shared with their classmates. Satisfaction in this mode decreased, however, if it was caused by unsuccessful problem-solving processes.

The final model describing the mastery of doing and learning mathematics in the Classpad project is presented in Fig. 2.

As the model shows, mastering doing and learning mathematics was the outcome of a complicated process shaped by the students’ conceptions of themselves as math learners, self-guided work style, minimalist instruction as a teaching strategy, and expertise in the project’s new technical tool, work style, and math content. The process was iterative, including shifts between modes. The typical students’ processes based on the model can be described by two profiles called the V and L profiles (see Fig. 3). The process of mastering doing and learning math that most of the students (n=21) went through during the project is illuminated by the V profile and the process of two students is described by L profile (n=2).

The V-profile students acquired expertise in all three (tool, work style, and content) subcategories and were satisfied. Typically, the students’ process for mastering these categories was as follows. In the beginning, the students were excited about the project, as learning math was very different compared to regular math lessons; students were asked to use a new tool and complete self-guided work. After the first few lessons, the satisfaction of the V-profile students collapsed as they faced difficulties in mastering a tool and work style that slowed down the process of solving the assigned math problems. When the project continued, the V-profile students acquired expertise in
the tool and later on in the work style. Expertise in the content, that is, solving the assigned math problems, was the most difficult to acquire. It usually took several lessons. As the V-profile students understood for the first time how they were able to solve problems self-guided by using the calculator, acquiring expertise in the content accelerated because of the students’ increased awareness of the possibility of learning math by using the new tool and new work style. The final outcome of the process for the V-profile students was mastering doing and learning math and feeling satisfied.

The learning process of two students is described by the L profile. The L-profile students (B4 and G7) mastered doing and learning math, but were nevertheless dissatisfied at the end of the project. In the beginning of the project, both were as excited about the project as the V-profile students. However, as the project progressed, the L-profile students had difficulty acquiring at least one of the three fields of expertise. One of the students (B4) acquired expertise in the new tool and work style, but did not acquire expertise in content, that is, completing the problems assigned for the project. This was due to his decision to work alone. Thus, he wrote that some of the tasks were easy to solve using ClassPad, but some tasks were too difficult to solve alone. The other student (G7) decided to work alone as well. She also did not acquire expertise in the content. She wrote that she did all the math tasks, but she learned almost nothing during the project due to the new work style. However, she mentioned that she had learned the concept of slope, which was crucial in understanding the equation of a straight line. Thus, she had acquired expertise in the tool and content, but did not feel she had expertise in the work style. The outcome of the project for the L-profile students was mastering doing and learning math while being dissatisfied.

Discussion

The model presented in this article describes the processes of how students acquired the expertise needed to do and learn math. The students’ conceptions of themselves as math learners set the basis for the process needed for mastering doing and learning math. The process was shaped by the teacher’s MI and the students’ expected role as self-guided and collaborative learners. Technology as a tool and problem-solving as the content were additional important elements of the process. Thus, the learning processes during the lessons were not traditional, i.e., led by the teacher but the students became student-centered learners.

The results of this study are in line with the results of many previous studies on SCL. The students experienced collaborative SCL as a very powerful method for solving math problems; the majority of the students were satisfied with their learning (V-type students), even though the teacher’s instruction was minimal (cf. Burnard 1999; Gibbs 1995; Neumann 2013; Zimmermann 2003; Wu and Huang 2007). The project’s cognitive results, i.e., good test scores (Eronen & Haapasalo 2006; Eronen & Haapasalo 2011) confirmed that the students’ conceptual refinement of
a linear function actually took place in addition to the students’ positive experiences of the learning process. The easy-to-use technology (e.g. calculator), and the problem buffet (e.g. variety of problems with different difficulty level) the teacher provided served as a broad and adaptive frame for self-guided learning that facilitated the students’ deeper understanding of the math problems compared to standard ordinary math lessons (cf. Järvelä, Hurme and Järvenoja 2011; Neumann 2013; Resta and Laferrière 2007). Finally, as many of the students described, the joint discussions and the opportunity to solve problems with peers and the teacher gave the students real moments of clarity that were, according to the students, not typical during regular teacher-centered math lessons (cf. Brandes and Ginnis, 1986; Harden and Crosby 2000; Kember 1997; Lea, Stephenson and Troy 2003; Neumann 2013).

The need for understanding and restructuring teaching in mathematics is very topical, as traditional teacher-centered teaching practices are still widely used in math education. Previous research (e.g., Genlott and Grönlund 2016; Francisco 2013; McCrone 2005; Sfard 2001) indicates that collaborative learning methods could improve learning processes and outcomes and increase learners’ abilities in problem-solving, administering, and memorizing mathematical concepts and principles. In addition, collaborative learning seems to foster resourcefulness and advanced reasoning abilities (Derting and Ebert-May 2010), as discussion allows students to test their ideas, to hear and incorporate others’ ideas, and to consolidate their thinking by putting their ideas into words, thus building a deeper understanding of key concepts (McCrone 2005). Discussion can also encourage students to take a more reflective stance on their mathematical reasoning to learn to communicate mathematically and to participate in a wider range of mathematical argumentation (Lampert and Cobb 2003; Sfard 2001). The model described in this article can be used as a tool for structuring and guiding learning situations toward SCL.

The findings of this study, however, indicate that the practical implementation of SCL is not simple, as learners’ and teachers’ expectations and motivations regarding the goals, implementation, and outcome of the process can vary. Thus, according to the results of this study, the new work style, MI, and the use of technology did not provide completely satisfying experiences for some students (L-type students). This finding is important to acknowledge, as feelings and affects are important, particularly in math learning (Evans, Hannula, Zan and Brown 2006; Leder, Pehkonen and Törner 2002). For example, Goldin (2002) stated that, traditionally, research on students’ performance in math has concentrated primarily on cognition, less on affect, and still less on interactions between the two. Goldin (2002) also stressed the following: “When individuals are doing mathematics, the affective system is not merely auxiliary to cognition – it is central” (60). Once the need to take affect into account is acknowledged, “the most important problem for research on affect in mathematics is the understanding of the interrelationship between affect and cognition” (Zan, Brown, Evans and Hannula 2006, p. 117).
The critical elements of the SCL process must be captured and understood in order to take the interrelationship between affect and cognition into account and to maximize the benefits of learning situations for all learners. According to this study’s findings, a successful learning process includes different modes, and it shifts between these modes. During the process, the learner faces satisfying and dissatisfying moments, which are crucial for gaining a thorough understanding of the task to be learned. According the students in this study, their feelings of satisfaction were stronger than their feelings of dissatisfaction, particularly after the students had been dissatisfied during the problem-solving process, and had a positive impact on the students’ overall interest in doing and learning math.

The students’ increased satisfaction after dissatisfying moments, as well as their strong positive experiences during the project, can be interpreted through intrinsic and extrinsic motivation (cf. Turner 1995; Webb 2009). Self-guided learning and teamwork with an appropriate tool and MI increased students’ intrinsic motivation toward learning math, whereas the traditional math lessons, in which the motivation is more extrinsic due to the teacher’s active role, were less appealing to them. In addition, strong “Aha” moments were powerful learning experiences for students, strengthening their intrinsic motivation toward learning math and their identity as math learners.

The model presented in this article describes the processes of how students acquired the expertise needed for mastering doing and learning math. It is relevant to ask how the validity of the model is confirmed. As the model was constructed by using the grounded theory method, the validity is evaluated using the four properties of grounded theory introduced by Glaser and Strauss: fitness, understanding, generality, and control (1967, p. 237).

The model of the math learning mastery process was constructed from the written data of 23 junior high school students who participated in a ClassPad project in mathematics. Thus, the validity of the model fits the substantive area, that is, math lessons where calculators, SCL, and MI are used, which is one of the criterion for evaluating the model created by the grounded theory method.

According to Glaser and Strauss (1967), laypeople should also be able to understand the model, it should be sufficiently general to be applicable to a multitude of diverse daily situations within the substantive area, and it should allow for partial control over the structure and process of daily situations in the substantive area. These three criteria were taken into account when the model was constructed; The model is described in detail by providing examples from the raw data, the model is general enough to be used in the school context in other subject areas, and the model provides information that can be used to plan and implement successful SCL processes in other contexts. However, according to Glaser and Strauss (1967), a model created by grounded theory is never finished. Even after the last line is written, the researcher often finds himself or herself elaborating and amending the theory, knowing more now than when the research was formally
concluded (Glaser and Strauss 1967, p. 256). This is the case with the model introduced in this article; it serves as an interesting starting point for further research.

The study has some limitations. The number of students participating in the study was small, and thus, the amount of data collected was small. However, it is typical in the grounded theory method that even a small amount of data can be turned into a validated model if the analysis is done carefully. The grounded theory method itself has been criticized as a relevant qualitative data analysis method (e.g., Goldthorpe 1997; Thomas and James 2006). According to Thomas and James (2006), the procedural rules make the grounded theory method an attractive approach, as it seems to offer a map for conducting a qualitative inquiry. However, they further asserted that researchers who pick up grounded theory risk losing the best of qualitative inquiry. In this research, the grounded theory method provided a suitable method for analyzing the students’ experiences and enabled the building of a model of mastery of learning and doing mathematics via the ClassPad project.

The application of the model more broadly, however, requires future development and testing in other contexts and environments. For example, the model introduced in this article is based on the learners’ experiences on student-centered learning session and on the content of the data collected for the study. Thus, the model illuminates the processes of acquiring expertise from the students’ point of view leaving the teacher’s role peripheral. This does not mean that the teacher’s role is insignificant during student-centered learning. Instead, it would be very important to develop the model in the future by discovering how the teacher can successfully facilitate the acquirement of expertise during student-centered learning sessions.

**Conclusion**

The model presented in this article not only describes the process of mastering doing and learning math, but can also be used to construct SCL situations. According to the model, the key elements for SCL from the students’ point of view, are easy-to-use technologies, shared understanding of the work style, and diverse problems. First, acquiring the tools plays an important role in the mastery process, and thus, the tools need to be carefully selected to fit the task to be accomplished (cf. Haapasalo and Samuels 2011). The easy-to-use technology is crucial element, as the tools should not require too much work to master. The appropriate tool speeds up the learning process and can make it more fun for learners. It also allows for a self-guided method of doing tasks and solving problems.

Second, the teacher should make certain all students are aware of the aims of the SCL process and of the need for collaboration (Speck 2003). As the SCL process can be strange to students the teacher should encourage them to ask for help in acquiring the work style. It is also important to clearly define the teacher’s role so the students know that a teacher will not be teaching the content (Webb 2009), but instead will be guiding and supporting them if needed (cf. Chiu 2004).
This could imply, for example, to the composition of and collaboration in the groups. If allowed students often like to work with their peers rather than in groups defined by the teachers. Consequently, it is possible that the composition of the group will not be optimal for collaborative learning. In this case, the teacher needs to facilitate the collaboration towards learning goals by guiding and limiting the students’ activities in the group and by ensuring that every student has an opportunity to collaborate (cf. Webb 2009). In practice, this means that when interfering the activities of a group the teacher explains the reasons for interference and provides systematic and practical help for supporting students’ self-regulated learning and collaboration skills. The model introduced in this article, however, needs to be developed further to better include the teacher’s role as the facilitator of learning processes during student-centered session.

Third, selecting the problems or tasks for learners is the final crucial element in making SCL successful. They should include a wide range of choices in terms of level of difficulty. Tasks that are too simple kill the joy of discovery for learners, and tasks that are too difficult are overly challenging, particularly for learners who are not used to or comfortable with the collaboration needed in the SCL process (cf. Ferrari and Mahalingham 1998). In addition, students can set different goals for their mastery. Some students are satisfied with reaching the level of doing math, which might be a dissatisfying situation for a student who wants to learn mathematics (cf. Shalberg and Berry 2003). Therefore, it is important to select the problems so that also those students who might select only the easiest problems will be able to reach the minimum curriculum standards.

Fourth, in order to foster SCL in mathematics, more micro-analytic research on students’ learning processes during math lessons is needed. For example, research methods such as discourse analysis and conversation analysis could bring new perspectives on students’ interaction and conversations during student-centered and traditional learning situations. The results of those studies would increase knowledge on students’ need for support during learning processes. In addition, the findings of this article reveal that learning math, particularly via a student-centered approach, evokes different kinds of feelings toward mathematics and impacts students’ views on learning mathematics. Thus, investigating feelings and their appearance, particularly during problem-solving processes, could provide support for students who might be in the danger of losing interest in mathematics due to their negative feelings toward the subject.

Finally, the model introduced in this article is an attempt to unwrap the structure of complicated learning processes during student-centered learning and to introduce components related to the processes from the students’ point of view. In addition, the model will facilitate the understanding the students’ feelings of satisfaction and dissatisfaction during student-centered learning. Consequently, the model can be used as a tool to understand the components and feelings that related to student-centered learning and to develop pedagogical practices to support self-regulated learning.

References


<table>
<thead>
<tr>
<th></th>
<th>Pre-test</th>
<th>Test</th>
<th>Re-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average total</td>
<td>16.5</td>
<td>42.4</td>
<td>39.4</td>
</tr>
<tr>
<td>Cronbach's Alpha</td>
<td>0.389</td>
<td>0.786</td>
<td>0.884</td>
</tr>
<tr>
<td>Pre-test</td>
<td>0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test</td>
<td></td>
<td>0.383</td>
<td></td>
</tr>
</tbody>
</table>

a) Average total test scores out of 60
b) Cronbach's alpha between test items during each test round
c) The significance of sign test for two related samples between pre-test and test
d) The significance of sign test for two related samples between test and re-test
<table>
<thead>
<tr>
<th>Categories and subcategories: mode of doing (D) and learning (L)</th>
<th>Tools in math lessons (T)</th>
<th>Working style in math lessons (W)</th>
<th>Content in math lessons (C)</th>
</tr>
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<tbody>
<tr>
<td>Conception of oneself as a student in math lessons</td>
<td>D  able to use tools in math lessons</td>
<td>able to work normally in math lessons</td>
<td>able to do content in math lessons</td>
</tr>
<tr>
<td></td>
<td>L  able to learn with tools in math lessons</td>
<td>able to learn math normally in math lessons</td>
<td>able to learn with content in math lessons</td>
</tr>
<tr>
<td>Self-guidance in math lessons</td>
<td>D  using tools self-guided</td>
<td>working self-guided</td>
<td>doing content self-guided</td>
</tr>
<tr>
<td></td>
<td>L  learn to use tools self-guided</td>
<td>learn to work self-guided</td>
<td>learn to do content self-guided</td>
</tr>
<tr>
<td>Minimal Instruction environment for math lessons (MIL)</td>
<td>D  using tools during MIL</td>
<td>working during MIL</td>
<td>doing content during MIL</td>
</tr>
<tr>
<td></td>
<td>L  learn to use tools during MIL</td>
<td>learn to work during MIL</td>
<td>learn to do content during MIL</td>
</tr>
<tr>
<td>Acquisition of expertise</td>
<td>D  succeeding to use tools self-guided during MIL</td>
<td>succeeding to work self-guided (during) MIL</td>
<td>succeeding to do content self-guided during MIL</td>
</tr>
<tr>
<td></td>
<td>L  succeeding to learn with tools self-guided during MIL</td>
<td>succeeding to learn self-guided (during) MIL</td>
<td>succeeding to learn with content self-guided during MIL</td>
</tr>
<tr>
<td>Mastering process with or without feeling of satisfaction</td>
<td>D  Mastering or not mastering doing concerning tools</td>
<td>Mastering or not mastering doing concerning working style</td>
<td>Mastering or not mastering doing concerning content</td>
</tr>
<tr>
<td></td>
<td>L  Mastering or not mastering learning concerning tools</td>
<td>Mastering or not mastering learning concerning working style</td>
<td>Mastering or not mastering learning concerning content</td>
</tr>
</tbody>
</table>
Connect the picture to the correct arguments:

- Slope is 2/3
- Slope is 3/2
- Slope is 4/6
- Point (4,3) is part of the straight line
- Point (4,4) is part of the straight line
- The origin is part of the line

Draw a straight line with a slope of 3 that goes through the point (8,6)

GSM operator A offers free calls for a total fixed price of 17,90€ per month. Operator B does not charge any basic fee, but the cost for every call is 0,10€ per minute. Can you make a graphic representation to compare which of these offers might be the most suitable one for you?